CICAM
COLLABORATIVE INNOVATION CENTER OF ADVANCED MICROSTRUCTURES

理论物理
暑期学校

超导与超流

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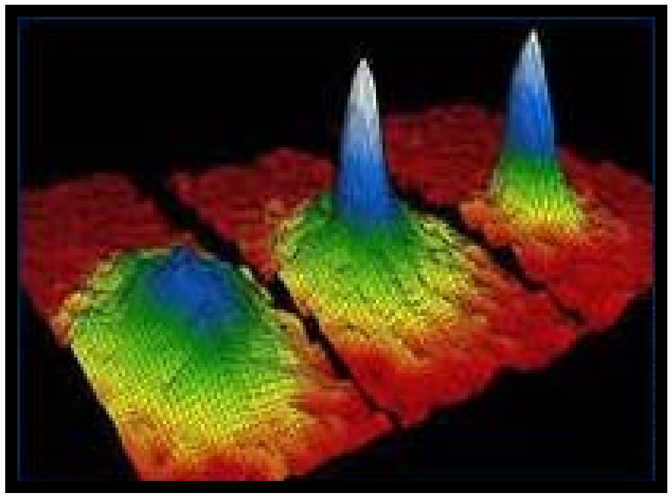
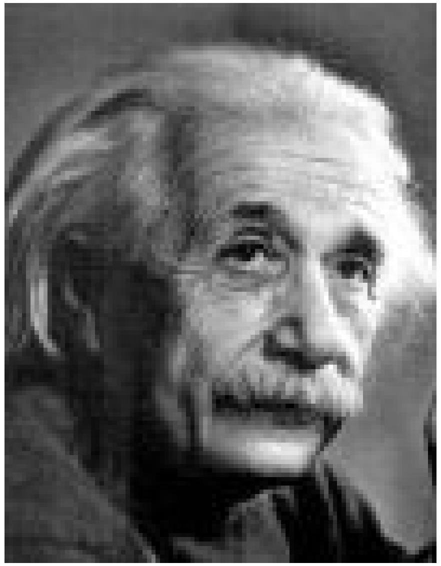
宏观量子现象

Superconductivity, superfluidity, and Bose-Einstein condensation are called **macroscopic quantum phenomena**. Their strange properties are direct consequences of quantum mechanics. That is why they only occur at low temperature.



全同粒子的关联现象：量子统计

S. N. Bose



A. Einstein

E. Fermi

P.A.M. Dirac

玻色统计：
每个状态可容纳任意多个粒子

费米统计：
每个状态最多可容纳一个粒子

Superconductivity/Superfluidity Associated with at Least Nine Nobel Prizes

- 1913 Onnes for superconductivity-expt
- 1972 Bardeen, Cooper, Schrieffer (BCS)-theory
- 1987 Bednorz and Muller– high Tc- expt
- 2001 BEC in trapped Bose gases-expt
- 2003 Abrikosov, Leggett, Ginzburg- theory
- 2008 Nambu– BCS theory in particle physics.

…….. and still counting !

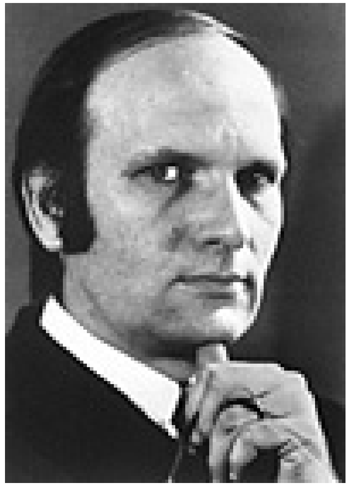
超导电性

1. Introduction to BCS Theory
2. From Hubbard model to t-J model
3. Plain-vanilla version of RVB theory
4. Numerical studies on t-J model

1. Introduction to BCS Theory





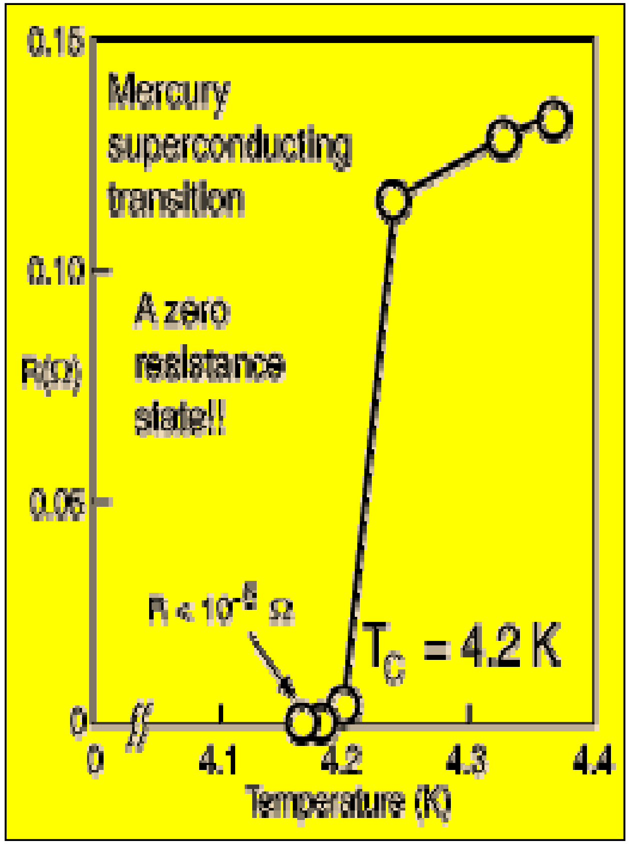


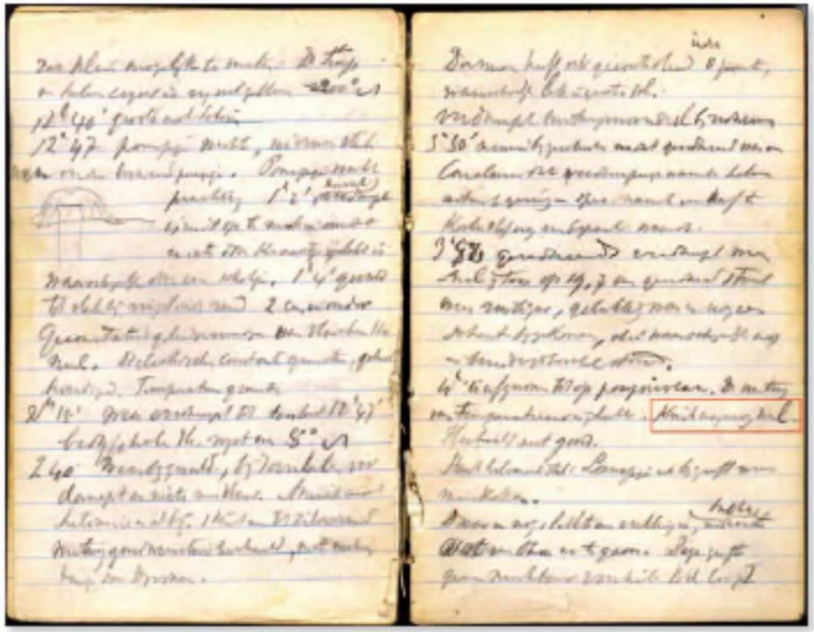

H. Kamerlingh Onnes (1913) John Bardeen Leon N. Cooper J. Robert Schrieffer (1972)

Superconductivity：Zero Electrical resistance

(Onnes,1911)

- 1902 Lord Kelvin: resistivity is infinity at 0K
- 1908 Onnes, succeeded in liquefying He





On April 8, 1911, K. Onnes worte in his notebook: "Mercurcury ('s resistance) is practically zero" at 3K.

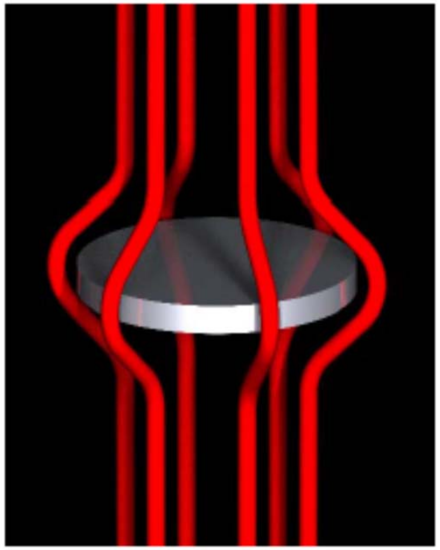
- 1914, Onnes discovered the persistent current phenomenon

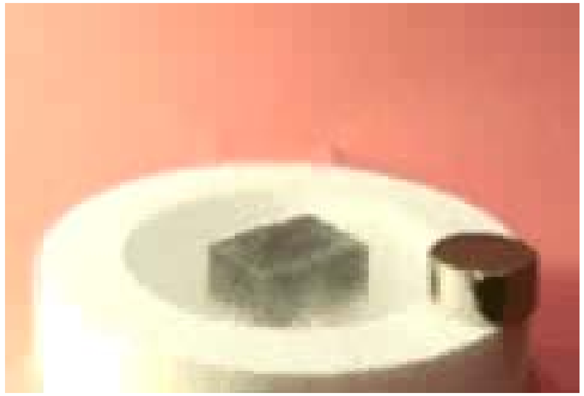
Superconductivity： Meissner Effect

Walther Meissner

Robert Ochsenfeld

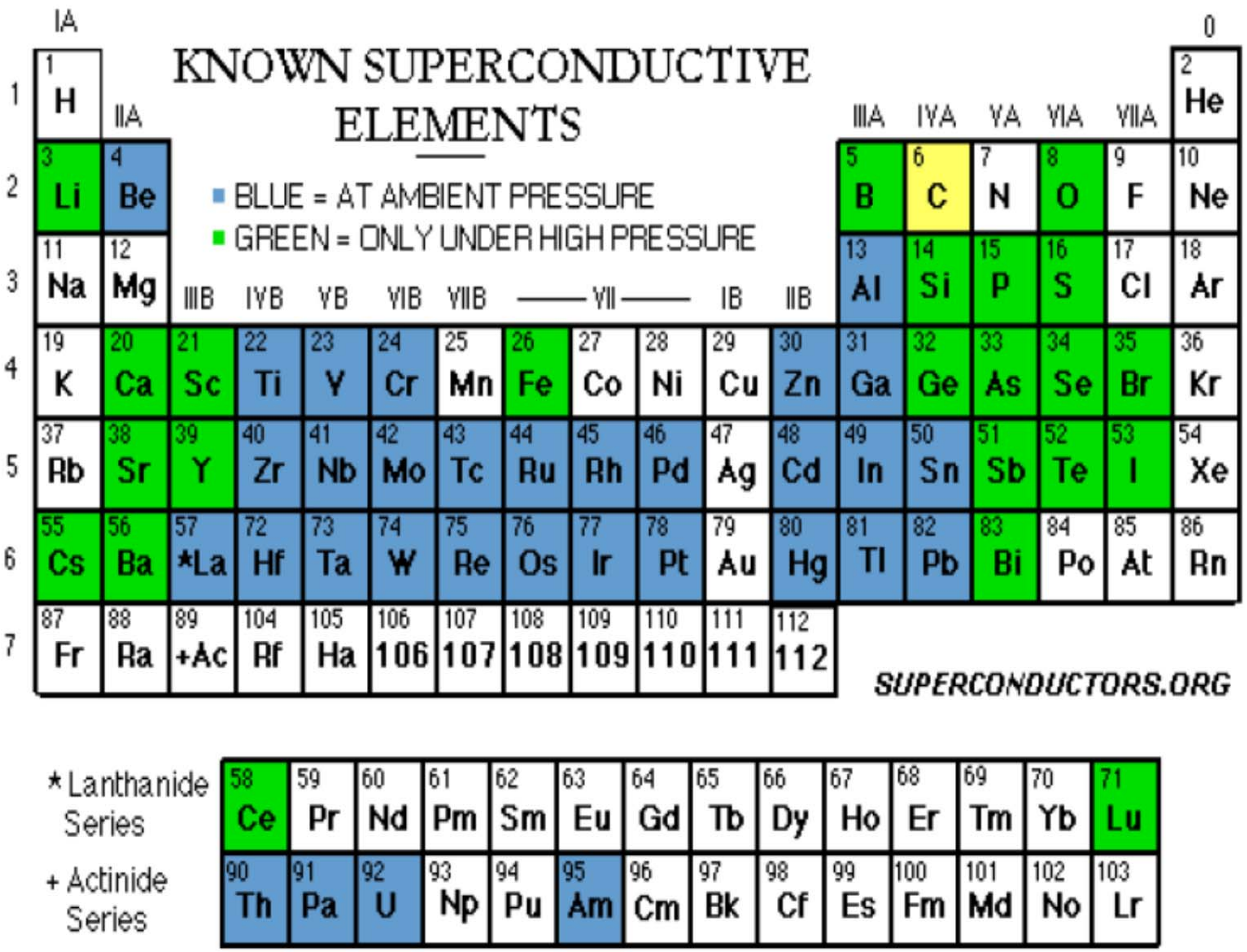
(1933)





- A superconductor can expel the magnetic flux inside.
- It is an effect that can not be derived from the perfect conductivity state (R=0).Another words, the Meissner effect is more than a phenomenon described by Lenz’s law.

Most single elements are superconductors

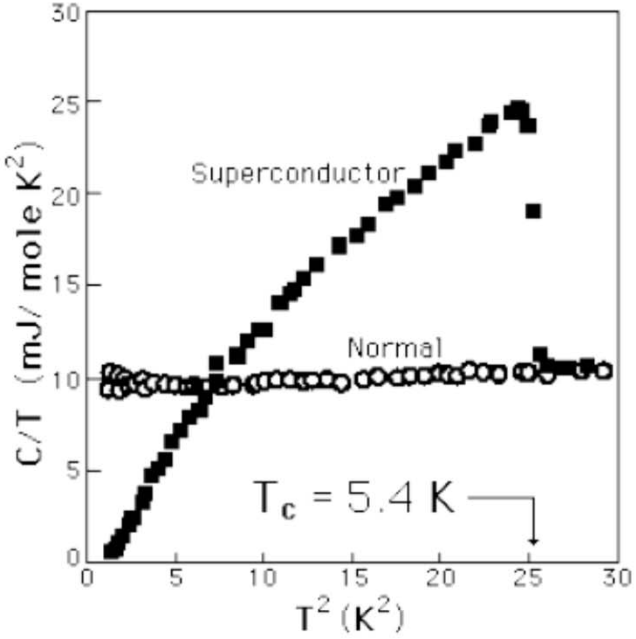


Element	T_c (K)
Nb	9.25
Pt	0.0019
Hg	4.2
Nb_3Sn	18
Nb_3Ge	23

Experimental Facts of Conventional Superconductivity

1) Perfect conductivity R = 0, and Meissner effect for $T < T_c$.

2) Specific heat jump at T_c and below



3) Isotope effect:

The isotope exponent in a property E
is defined as: $\alpha(E) = -(\Delta E/E)/(\Delta M/M)$

M is the isotope mass, E = T_c *,* T^**,* ρ_s *, - - -*

T_c = const. exp ($-\alpha$)

Zn	0.45 ± 0.05
Cd	0.50 ± 0.10
Sn	0.47 ± 0.02
Hg	0.50 ± 0.03
Pb	0.48 ± 0.01
Tl	0.50 ± 0.10
Al	
Ru	0.00 ± 0.05
Os	0.10 ± 0.10
Mo	0.37 ± 0.07

J.W. Garland Phys. Rev. Lett. **11**, 114 (1963)

Pre-BCS Macroscopic Theories of Superconductivity

Jorg Schmalian arXiv 1008.0447 (2010).:
Failed theories of superconductivity

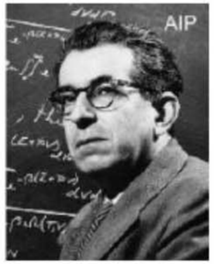
Albert Einstein
(1879-1955)

Niels Bohr
(1885-1962)

Ralph Kronig
(1905-1995)

John Bardeen
(1908-1991)

Werner Heisenberg
(1901-1976)



Fritz London
(1900-1954)

Lev D. Landau
(1908-1968)

Felix Bloch
(1905-1983)

Léon Brillouin
(1889 -1969)

Max Born
(1882-1970)

Herbert Fröhlich
(1905-1991)

Richard Feynman
(1918-1988)

An impressive assembly of prominent scientists worked on the origin of LT superconductivity during the period of 1911 to 1957, almost 50 years. Their contributions to the understanding of LTSC were quite valuable.

What did they learn?

Pre-BCS Macroscopic Theories of Superconductivity

Based on the observation of an isotope effect, it was concluded by Frohlich and Bardeen: electron-lattice coupling must be involved in forming a SC state.

Based on the temperature dependence of specific heat

Feynman remarked that; " - - - since the transition temperature corresponds to an energy 'kT = 10-4 ev, while electron energies are of the order 10ev or so.' The model couldn't be just an ideal gas of electrons, because that doesn't give superconductivity, but it couldn't be too far different either. It couldn't be due to the Coulomb interaction because that's much too strong, 'a volt or so per electron'.

Based on the sub-linear temperature dependence of specific heat

Feynman knew that: '. . . that the ground state is separated from the higher excited states by a region where the density of states is low,' or it may even be the case that there are 'no states in between' In other words, it means there is an energy gap above the ground state in the energy spectrum.

Conventional Superconductivity: Three major insights of BCS Theory

Bardeen

Cooper

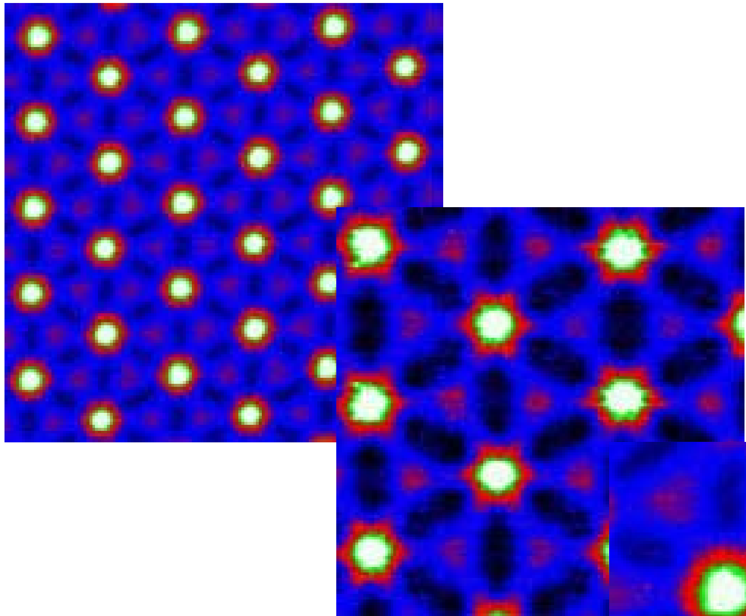
Schrieffer '57

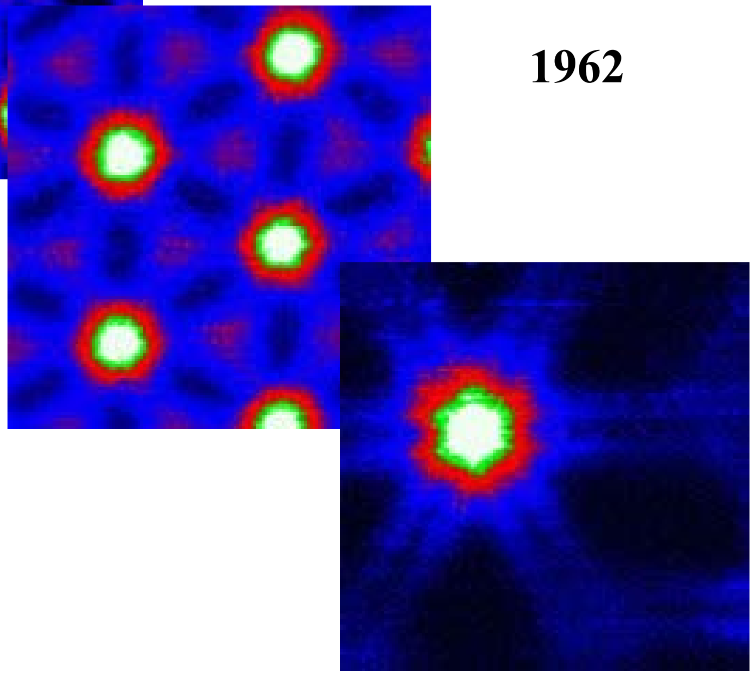
The Nobel Prize in Physics 1972

(15 yrs after BCS theory published)

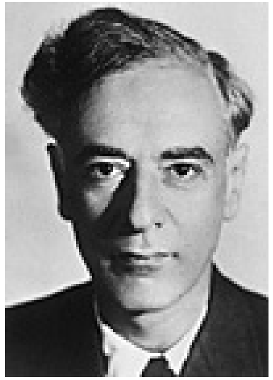
- The effective forces between electrons can sometimes be attractive in a solid rather than repulsive
- "Cooper Pair"::two electrons outside of an occupied Fermi surface form a stable pair bound state, and this is true however weak the attractive force
- Schrieffer constructed a many-particle wave function which all the electrons near the Fermi surface are paired up.

Landau-Ginzberg Theory
and Quantized Magnetic Flux Vortices



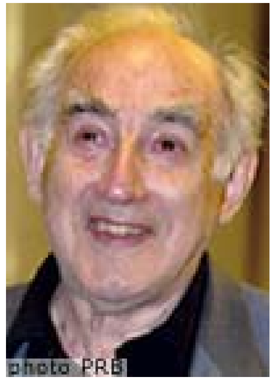


Vortices of $NbSe_2$



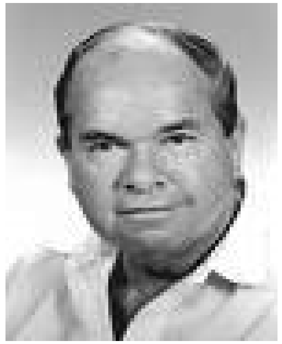
Landau

1962



Ginzberg

2003



A. A. Abrikosov

2003 Nobel Prize

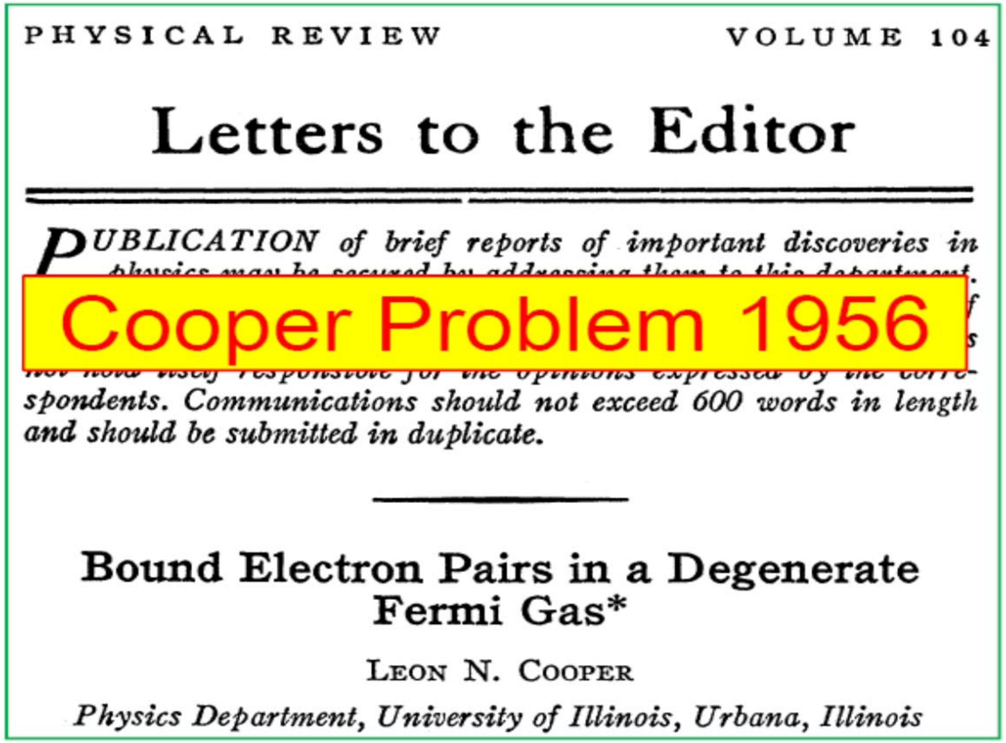
BCS Theory: Superconducting Mechanism

1. **What is the interaction driving electron to form pairs, and how?**
2. **How do Cooper pairs form phase coherence to condense?**
3. **How to describe the superconducting state**

BCS theory

- solve the first problem for "electron-phonon"-type superconductors
- the second problem is not important for superconductors whose superconducting phase fluctuation is weak
- solve the third problem comprehensively

BCS Theory : Cooper pair formation



PHYSICAL REVIEW VOLUME 104

Letters to the Editor

PUBLICATION *of brief reports of important discoveries in*

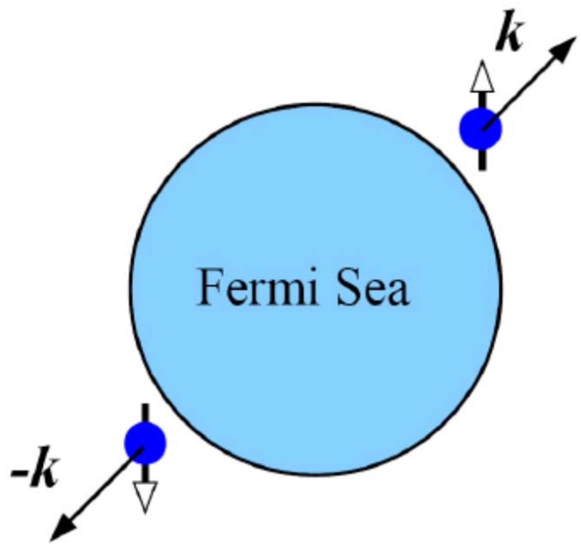
Cooper Problem 1956

spondents. Communications should not exceed 600 words in length and should be submitted in duplicate.

Bound Electron Pairs in a Degenerate Fermi Gas*

LEON N. COOPER

Physics Department, University of Illinois, Urbana, Illinois



If two electrons attract each other (V<0) in the presence of a Fermi sea then they always form a bound state, regardless how weak the attraction is

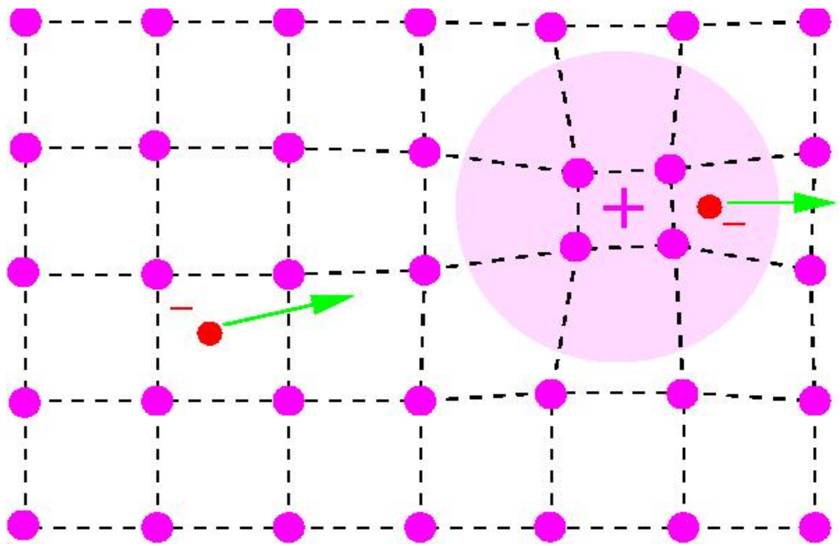
The bound state energy: $$\Delta = \hbar\omega_D e^{-2/|V|N_F}$$

DOS at Fermi level

BCS Theory: Pairing Glue?

- The interaction of the electrons with lattice vibrations (phonons) must be important (isotope effect)

Polarization of the lattice by one electron leads to an attractive potential for another electron



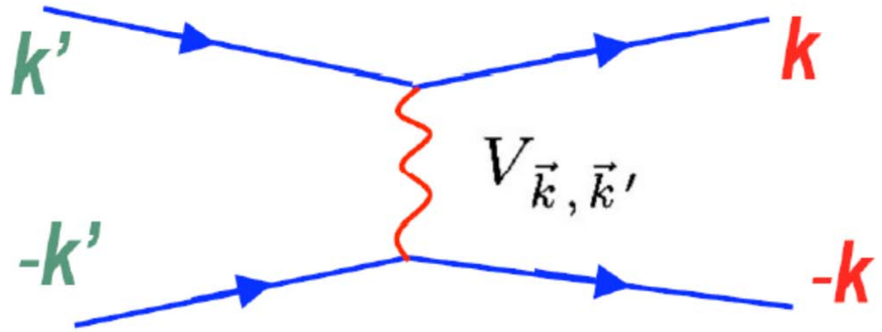
- The lattice atoms move much slower than the electrons. There is a time-retarded effective attraction between two electrons which glue electrons together

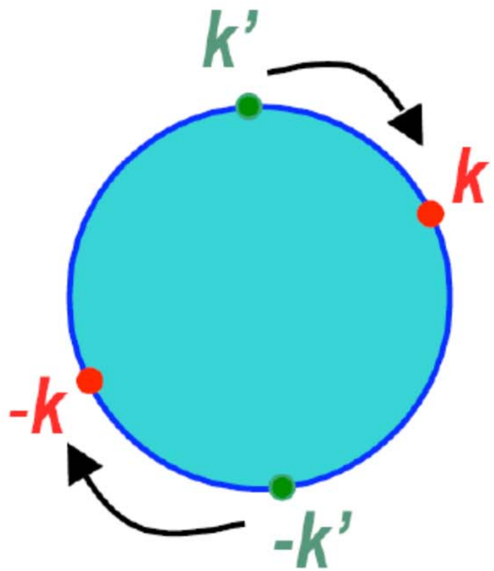
BCS Theory: Pairing Interaction

Cooper pair formation needs attractive interaction

$$\boxed{H_{pair} = \sum_{kk'} V_{k,k'} c^{+}_{k\uparrow} c^{+}_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}}$$

pair potential resulted from
electron-phonon coupling or any
other kind of interactions





BCS Theory: Model Hamiltonian

$$H = \sum_k \xi_k \left(c^+_{k\uparrow} c_{k\uparrow} + c^+_{k\downarrow} c_{k\downarrow} \right) + \sum_{kk'} V_{k,k'} c^+_{k\uparrow} c^+_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow}$$

Kinetic energy **Pairing Potential**

BCS mean field approximation

$$\begin{aligned} H_{MF} &= \sum_k \xi_k \left(c^+_{k\uparrow} c_{k\uparrow} + c^+_{k\downarrow} c_{k\downarrow} \right) + \sum_{kk'} V_{k,k'} \left(\left\langle c^+_{k\uparrow} c^+_{-k\downarrow} \right\rangle c_{-k'\downarrow} c_{k'\uparrow} + c^+_{k\uparrow} c^+_{-k\downarrow} \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle - \left\langle c^+_{k\uparrow} c^+_{-k\downarrow} \right\rangle \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle \right) \\ &= \sum_k \xi_k \left(c^+_{k\uparrow} c_{k\uparrow} + c^+_{k\downarrow} c_{k\downarrow} \right) - \sum_k \left(\Delta_k c^+_{k\uparrow} c^+_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow} - \left| \Delta_k \right|^2 \right) \end{aligned}$$

BCS mean field equation

$$\Delta_k = -\sum_{k'} V_{k,k'} \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle$$

BCS Theory: Gap Equation

BCS Mean Field Hamiltonian

$$H_{MF} = \sum_k \xi_k \left(c_{k\uparrow}^{+} c_{k\uparrow} + c_{k\downarrow}^{+} c_{k\downarrow} \right) - \sum_k \left(\Delta_k c_{k\uparrow}^{+} c_{-k\downarrow}^{+} + \Delta_k^{*} c_{-k\downarrow} c_{k\uparrow} - |\Delta_k|^2 \right)$$

Self-consistent equation

$$\Delta_k = -\sum_{k'} V_{k,k'} \left\langle c_{-k'\downarrow} c_{k'\uparrow} \right\rangle$$

$$\Big\downarrow \text{ Solving BCS mean field Hamiltonian}$$

$$\Delta_k = -\sum_{k'} V_{k,k'} \frac{\Delta_{k'}}{2\sqrt{\xi_{k'}^2 + \Delta_{k'}^2}} \tanh \frac{\sqrt{\xi_{k'}^2 + \Delta_{k'}^2}}{2T}$$

BCS theory explains all features of conventional superconductors

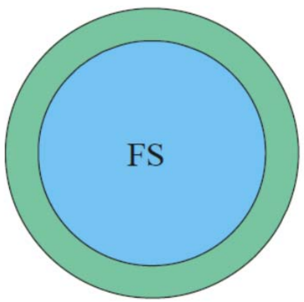
Symmetry of Cooper Pairs

Pair wavefunction: $$\Delta_{ss'}(\vec{k}) = \langle \hat{c}_{\vec{k}s}\hat{c}_{-\vec{k}s'} \rangle = \underbrace{\Phi(\vec{k})}_{\text{orbital}}\underbrace{\chi(s,s')}_{\text{spin}}$$

totally antisymmetric under electron exchange

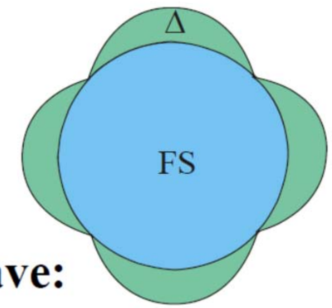
$$\vec{k} \rightarrow -\vec{k} \qquad s \leftrightarrow s'$$

even parity **L = 0,2,4,...**	$\Phi(-\vec{k}) = \Phi(\vec{k})$	$\longrightarrow$	**S=0 singlet**
odd parity **L=1,3,5,…**	$\Phi(-\vec{k}) = -\Phi(\vec{k})$	$\longrightarrow$	**S=1 triplet**



s-wave: $\Delta_k = \Delta$

d-wave:



$\Delta_k = \Delta(\cos k_x - \cos k_y)$

BCS Theory: Variational Wavefunction

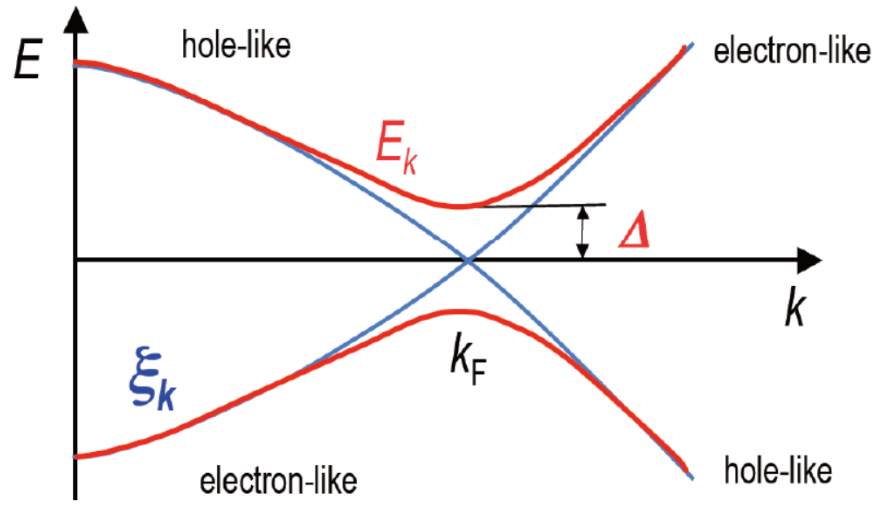
Macroscopic Coherent Pair Wave Function forms at T < Tc

$$\boxed{\left|\Psi_{BCS}\right\rangle = \prod_k \left(u_k + v_k c^+_{k,\uparrow} c^+_{-k,\downarrow}\right)\left|0\right\rangle}$$

Electron number is not conserving

probability in a Cooper state = $|v_k|^2$

Quasiparticle Spectrum： $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$



BCS Theory: Critical Temperature

s-wave superconductor

$$\boxed{T_c \approx 1.134\hbar\omega_D e^{-1/VN_F}}$$

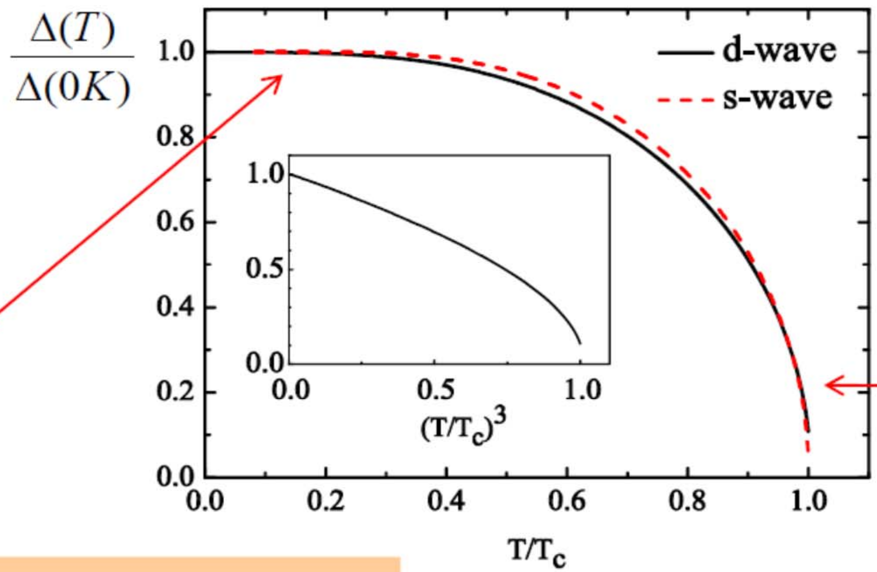
$$\boxed{\frac{\Delta(0K)}{T_c} \approx 1.76}$$

d-wave superconductor

$$\boxed{T_c \approx 1.134\hbar\omega_D e^{-2/VN_F}}$$

$$\boxed{\frac{\Delta(0K)}{T_c} \approx 2.14}$$

$\Delta(T) \sim T^3$



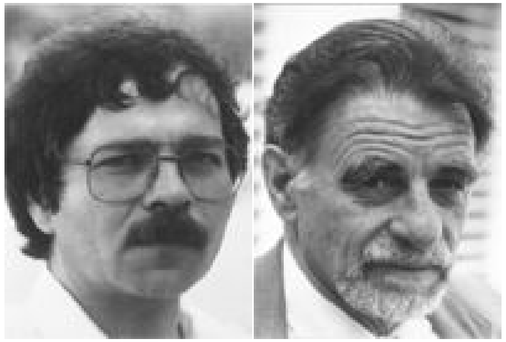
$\Delta(T) \sim \sqrt{T_c - T}$

$T_c \propto M^{-\frac{1}{2}}$

isotope effect !

2. From Hubbard model to t-J model

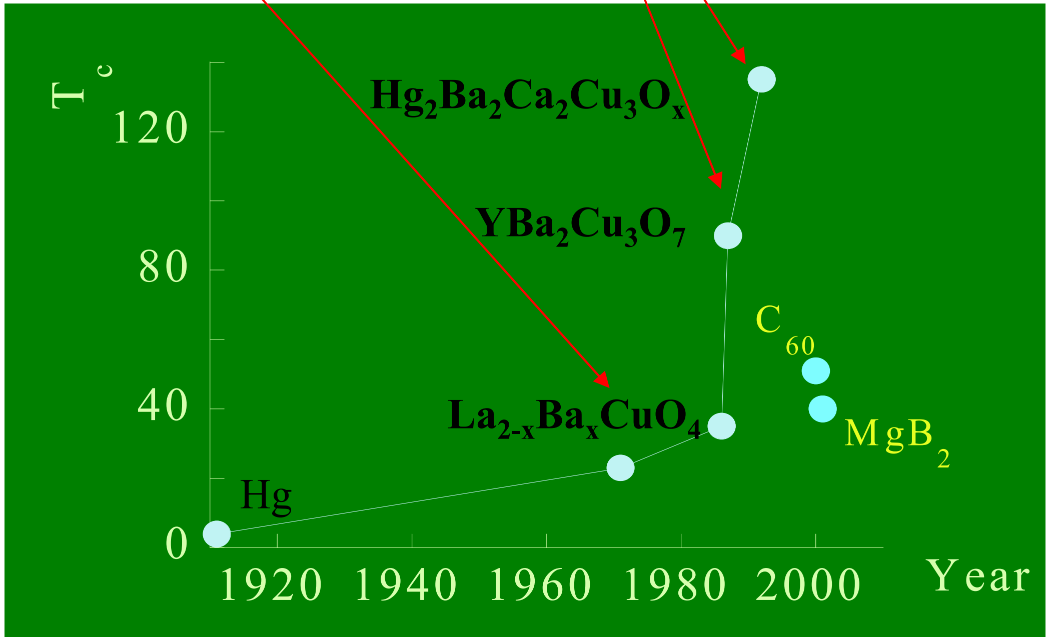
High-Tc Superconductors

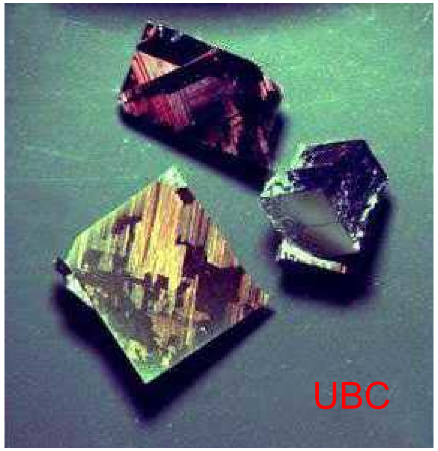


Bednorz & Muller 1986

P. Chu

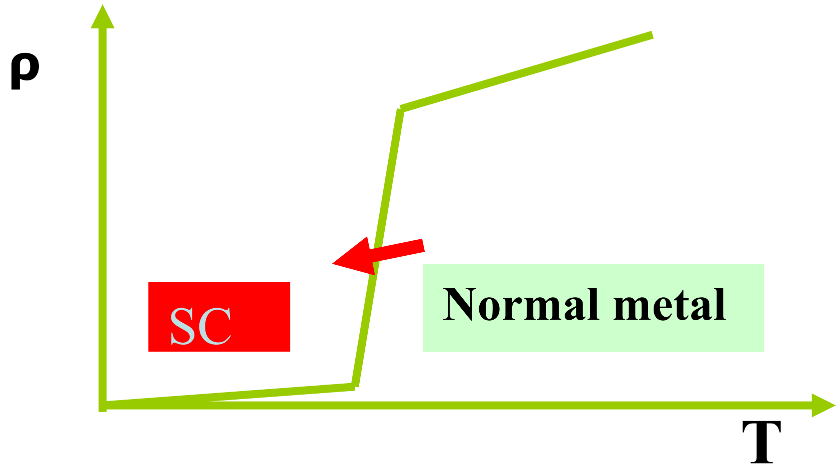
One of the most challenging problems left last century



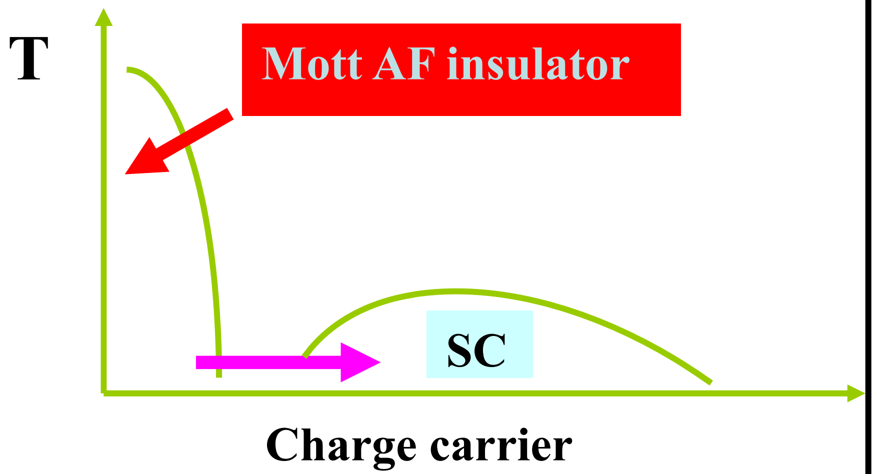


和而不同

Conventional Superconducitivity develops in metals (like mercury)



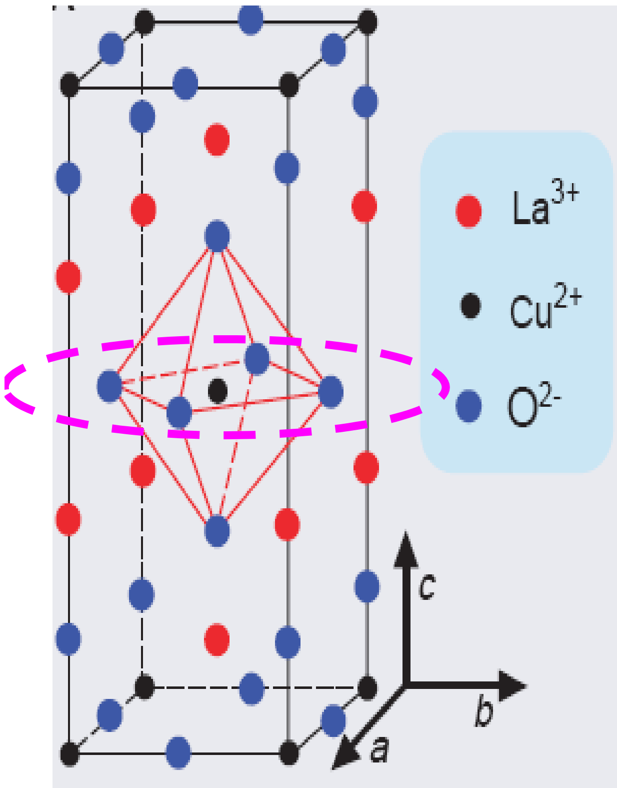
High Tc cuprate superconductivity develops out of a type of insulator called Mott insulator due to e-e interaction



High-Tc Materials Crystal Structure

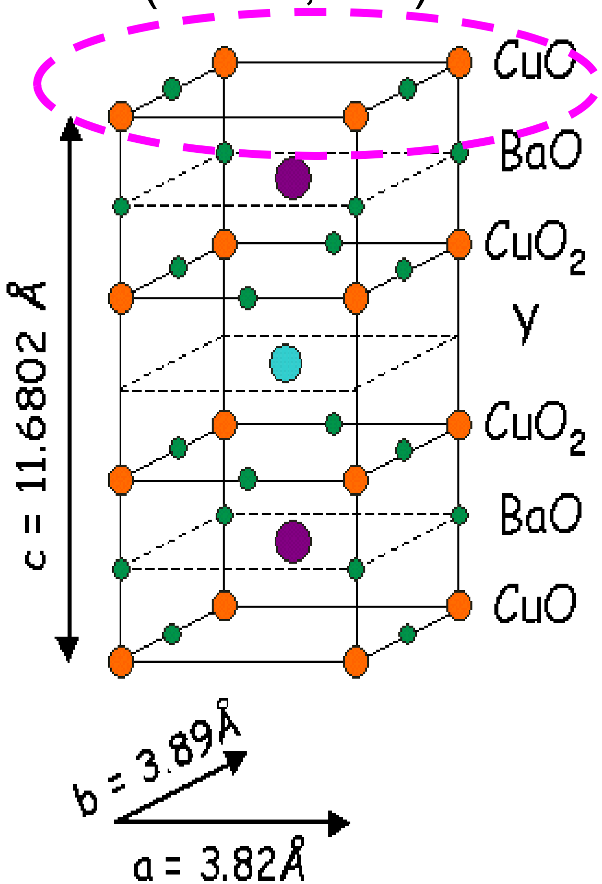
$La_{2-x}Sr_xCuO_4$

(Tc=40K)



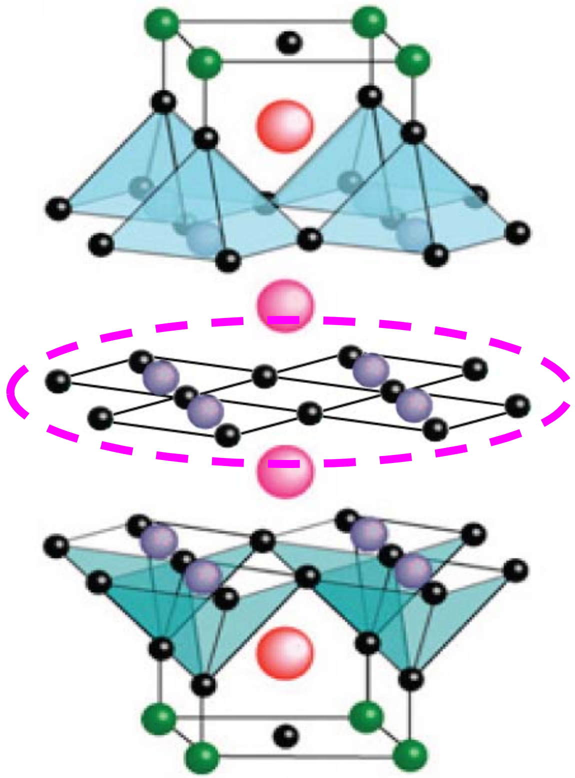
$YBa_2Cu_3O_{7-\delta}$

(Tc=60,90K)



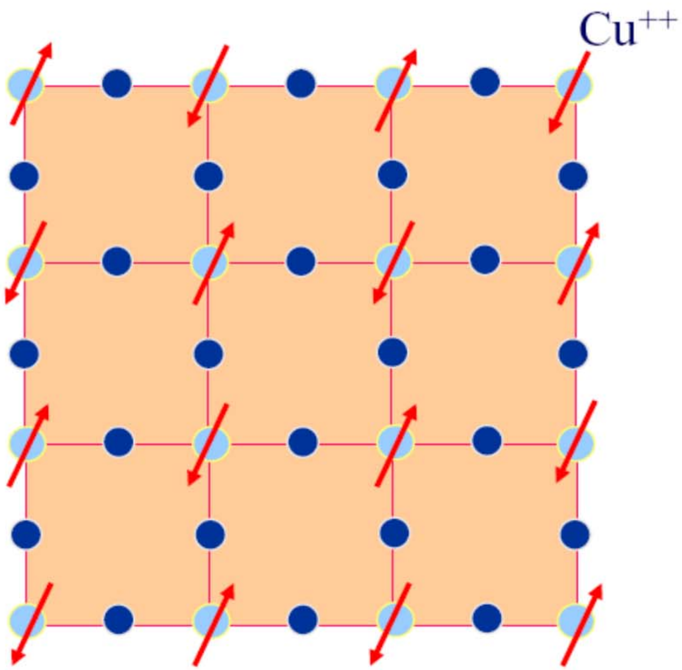
$Bi_2Sr_2Ca_{2-x}Cu_3O_{10+d}$

(Tc=120K)

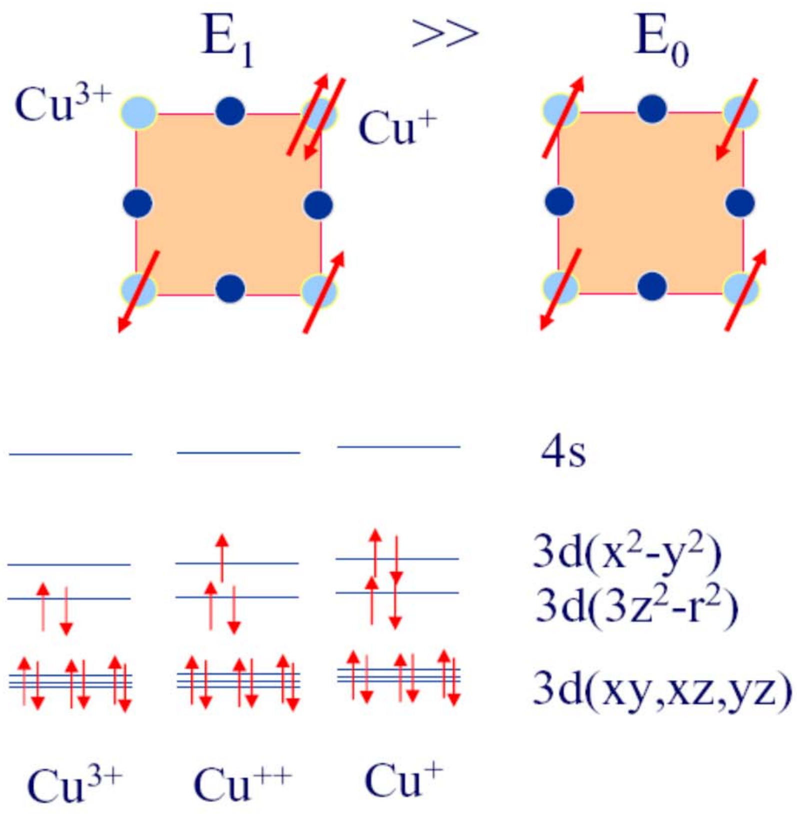


- **Two-dimensional layered structure**
- **Most important physics in the common CuO2 plane**

Parent Compound: AFM Mott Insulator



$T_N \sim 500K$

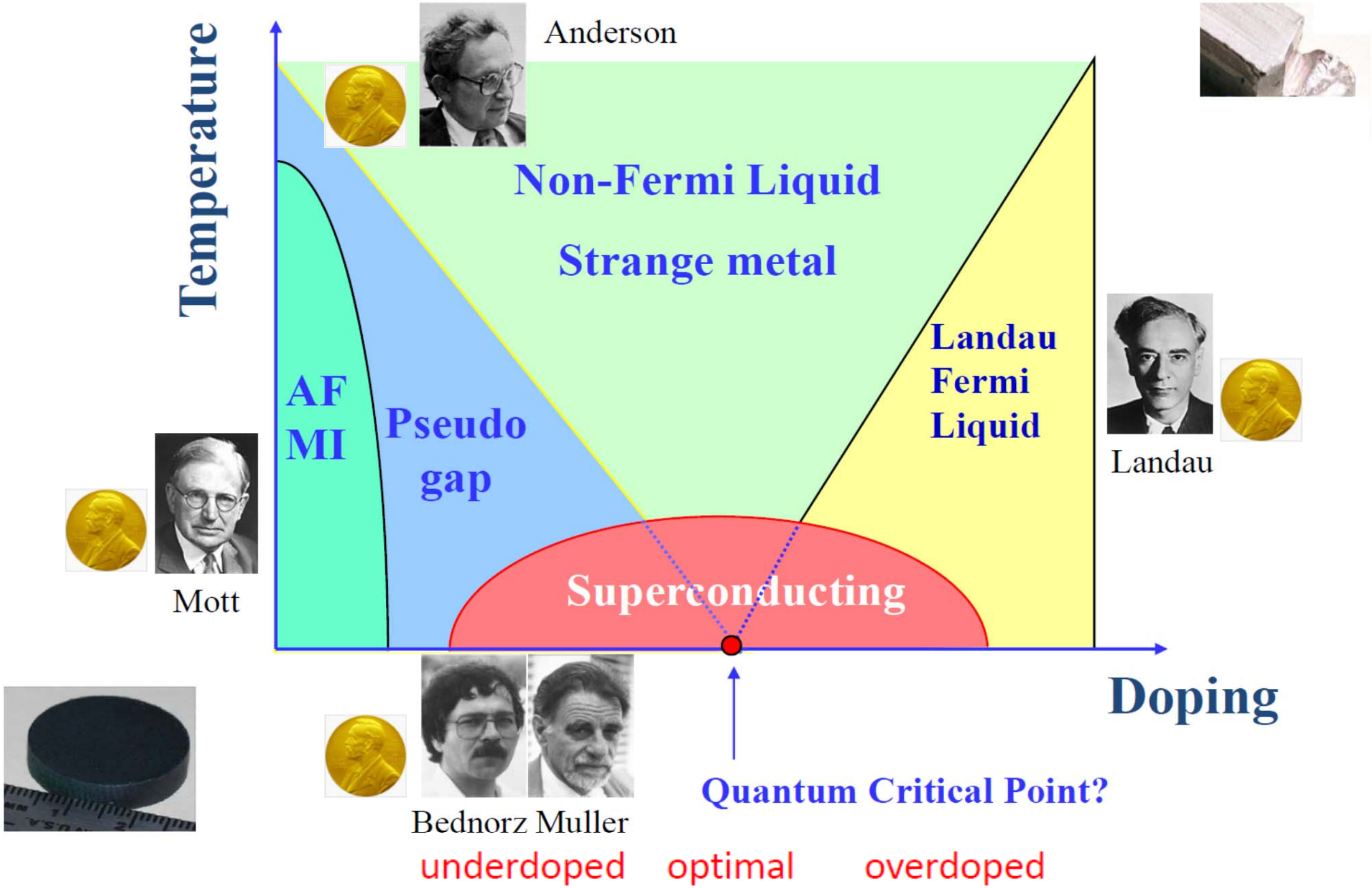


Low-energy physics purely
Due to electron spins

$$H_{Heisenberg} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

Doped Mott Insulator: Main Physics

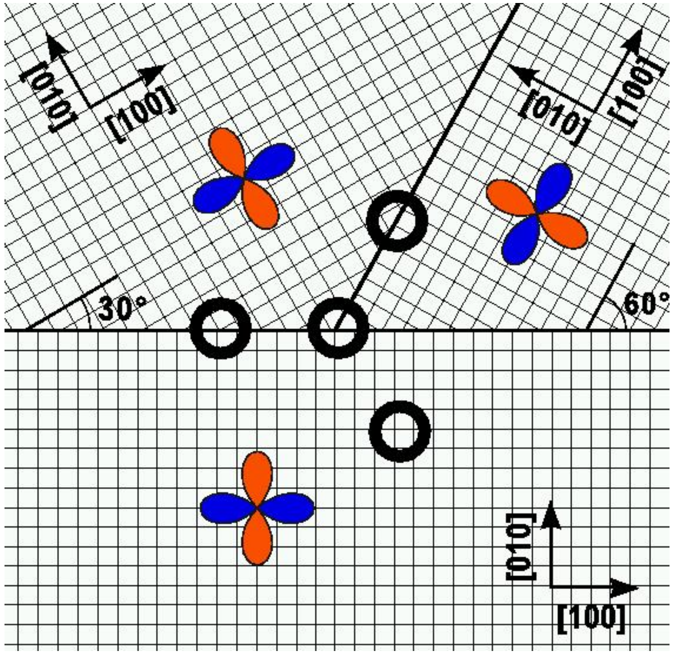
Phase Diagram： High-Tc Materials



Everything starts from doping a half-filled AF Mott insulator

Direct Detection of Pairing Phase Factor

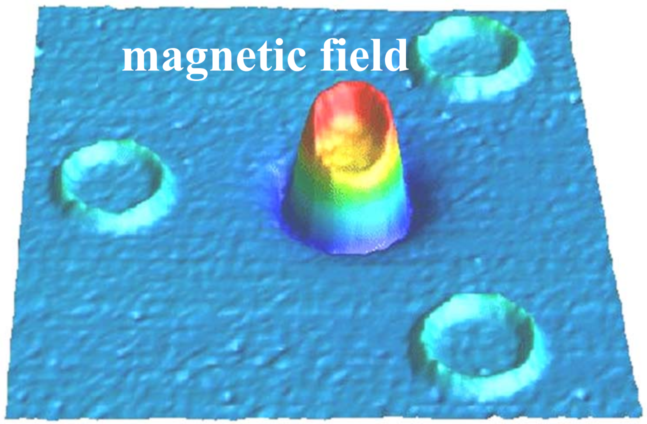
Tri-Crystal Geometry



Tsuei, Kirtley et al. (1995)

Odd number of pi-shifts

frustrated loops lead to a current in groundstate



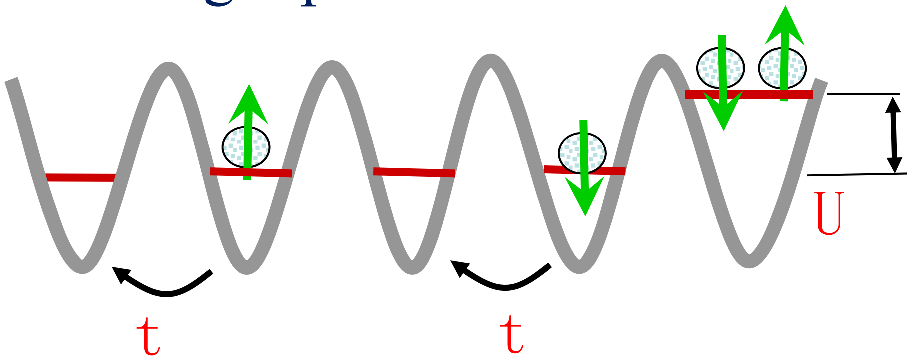
Superconducting loop
$YBa_2Cu_3O_7$ $T_c = 92$ K ϕ = 60 mm

C C Tsui 1998
Burkley Prize

Minimal Model for Single-band Description

- 2D square lattice system
- electron-electron interaction alone
- strong repulsive Coulomb interaction

$\Bigg\}$ **Hubbard model**

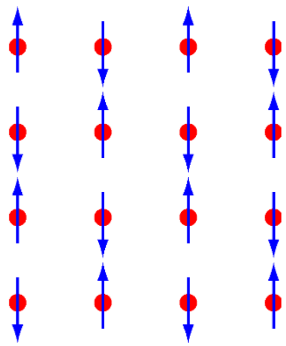


$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle} (c_{ia}^{\dagger} c_{ja} + \text{h.c.}) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

antiferromagnetism at half filling

(half filling = one electron per site = zero doping)

Heisenberg model
(*t*-*J* model)



Néel order

$$\langle \mathbf{S}_i \rangle = S_0 (-1)^{i_x + i_y} \, \hat{z}$$

Hubbard model(Phenomena predicted)

Superexchange and antiferromagnetism (P.W. Anderson)

Itinerant ferromagnetism. Stoner instability (J. Hubbard)

Incommensurate spin order. Stripes (Schulz, Zaannen, Emery, Kivelson, White, Scalapino, Sachdev, …)

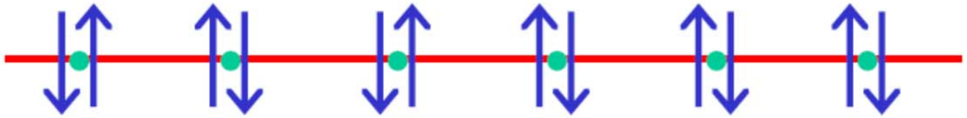
Mott state without spin order. Dynamical Mean Field Theory (Kotliar, Georges,…)

d-wave pairing (Scalapino, Pines,…)

d-density wave (Affleck, Marston, Chakravarty, Laughlin,…)

Basic Physics of Strong Correlation

Band (Pauli) Insulator
= Even number of e⁻ per site



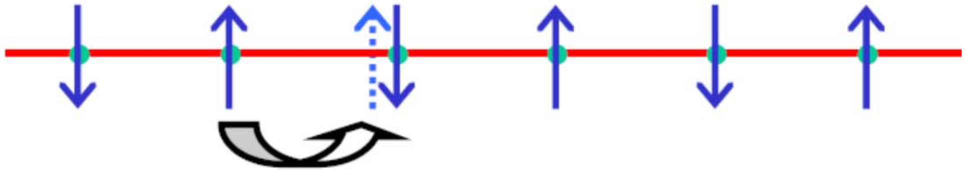
Mott Insulator
= Interaction driven insulator

half-filled case = one e⁻ per site

Hubbard Model

$$H = -t\sum_{\langle i,j\rangle}(c^{\dagger}_{i\sigma}c_{j\sigma} + h.c.) + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

$U =$ on-site repulsion



$t =$ hopping integral

Why is it difficult to solve?

$U >> t$: Mott Insulator

Single-site Hilbert space:

$|0\rangle$ $|\uparrow\rangle$ $|\downarrow\rangle$ $|\uparrow\downarrow\rangle$

Total number of states : lattice with N_s sites:

$$4^{N_s}$$

Example of 2-site Hubbard Cluster

$$H = -t(c_{1\uparrow}^{\dagger}c_{2\uparrow} + c_{1\downarrow}^{\dagger}c_{2\downarrow} + c_{2\uparrow}^{\dagger}c_{1\uparrow} + c_{2\downarrow}^{\dagger}c_{1\downarrow}) + U(n_{1\uparrow}n_{1\downarrow} + n_{2\uparrow}n_{2\downarrow})$$

Total number of states=4^2=16,   H is a 16x16 matrix, use symmetries
S_z is a good quantum number, H block diagonal for different S_z

Consider subspace with total S_z=0: 4 states

$$|\psi_1\rangle = |\uparrow_1;\downarrow_2\rangle$$

$$|\psi_2\rangle = |\downarrow_1;\uparrow_2\rangle$$

$$|\psi_3\rangle = |\uparrow\downarrow_1;0_2\rangle$$

$$|\psi_4\rangle = |0_1;\uparrow\downarrow_2\rangle$$

$$\langle\psi_\alpha|H|\psi_\beta\rangle = H_{\alpha\beta} \qquad H = \begin{pmatrix} 0 & 0 & -t & -t \\ 0 & 0 & -t & -t \\ -t & -t & U & 0 \\ -t & -t & 0 & U \end{pmatrix}$$

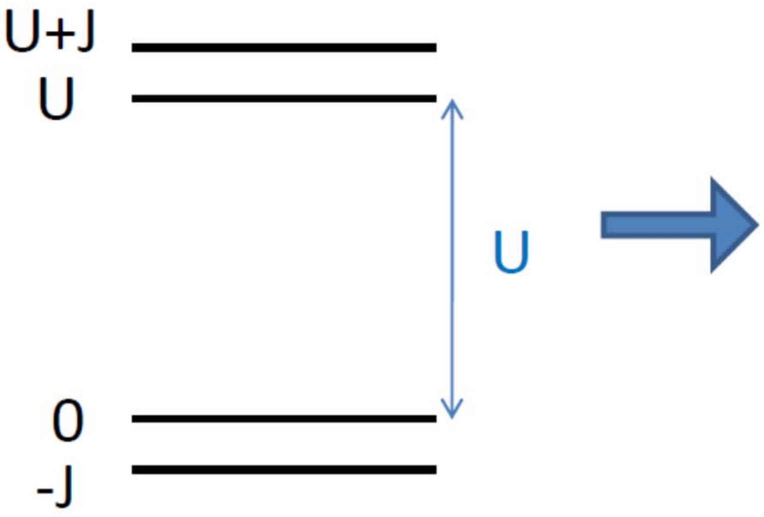
Eigenvalues:

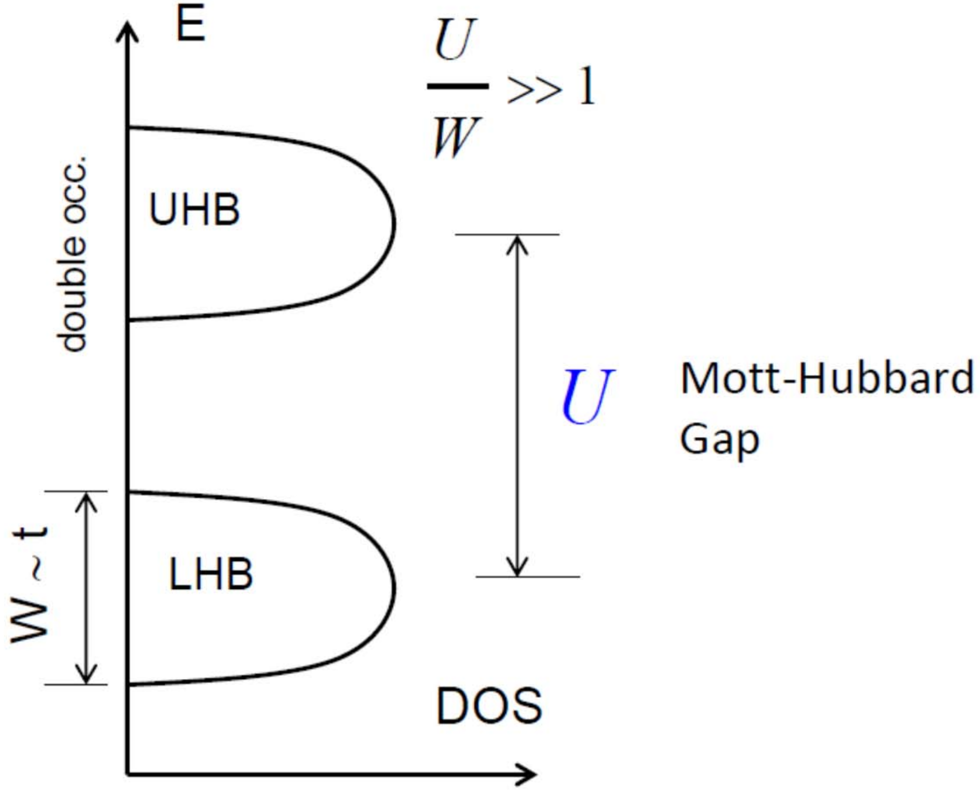
$$= \frac{1}{2}(U - \sqrt{U^2 + 16t^2}), 0, U, \frac{1}{2}(U + \sqrt{U^2 + 16t^2})$$

$$\approx -J,\ 0,\ U,\ U+J \ \text{ when } U >> t.\ J = \frac{4t^2}{U}$$

Upper and Lower Hubbard Bands

Two sites





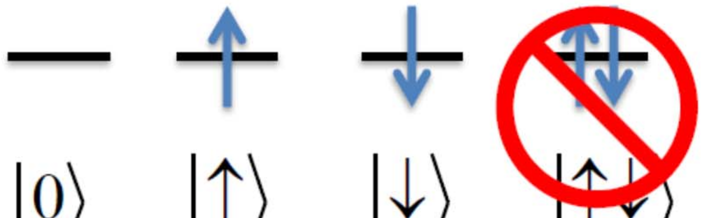
Lowest energy state:

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow_1;\downarrow_2\right\rangle-\left|\downarrow_1;\uparrow_2\right\rangle\right)$$

S=0 Spin singlet bond

Large U： Projection of Upper Hubbard Band

For large *U>>t,* project out doubly occupied sites



$|0\rangle \quad |\uparrow\rangle \quad |\downarrow\rangle \quad |\uparrow\downarrow\rangle$

Virtual hopping favors AFM correlations

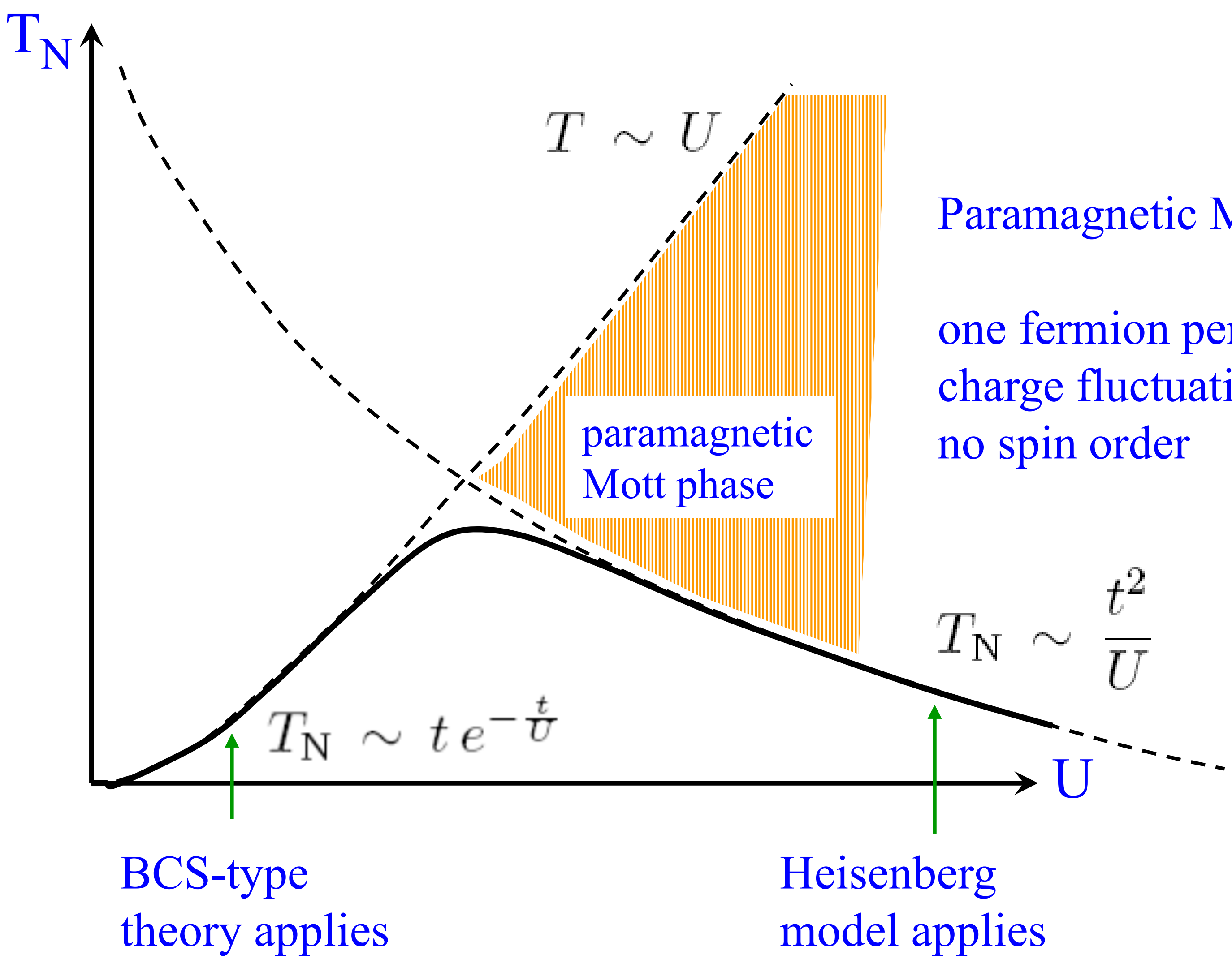
Canonical transformation → t-J model in projected Hilbert space

$$H_{t-J} = -t\sum_{\langle i,j\rangle} P_G(c_{i\sigma}^{\dagger}c_{i\sigma} + h.c.)P_G + J\sum_{\langle i,j\rangle}(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4}\hat{n}_i\hat{n}_j)$$

AF superexchange: $J \approx \dfrac{4t^2}{U} > 0$

No double occ. constraint: $\sum_{\sigma} c_{i\sigma}^{\dagger}c_{i\sigma} = 0,1$

Hubbard model at half filling



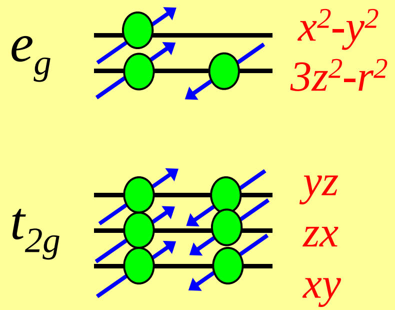
Paramagnetic Mott phase:

one fermion per site
charge fluctuations suppressed
no spin order

Parent Compound: AFM Mott Insulator

Cuprates => CuO_2 plane electronically relevant

Cu^{2+} *3d*^{9}



1 hole in 3d-e_g

spin S = 1/2

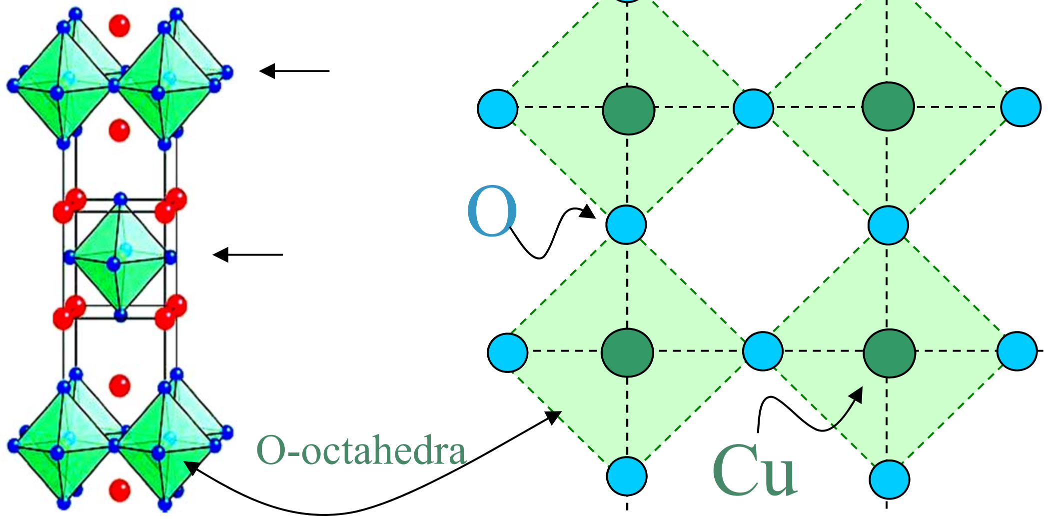
La_2CuO_4

CuO_2-plane

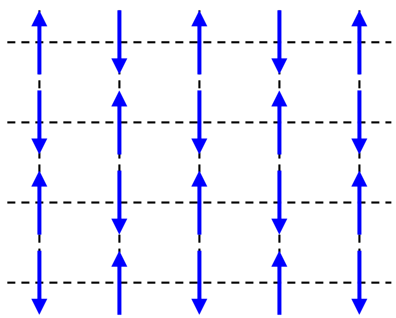
2D copper oxide

weak interlayer coupling

2D copper oxide



square lattice of Cu^{2+}-ions



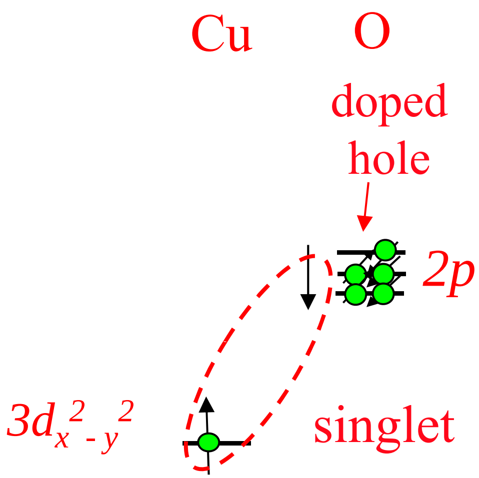
Strong Coulomb interaction

⇓

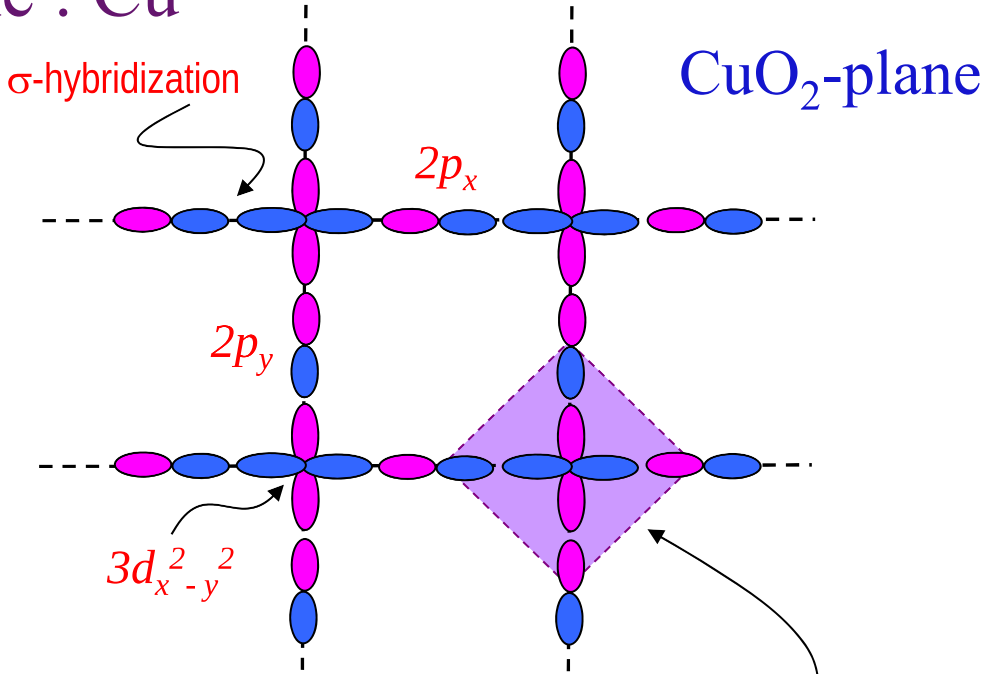
Mott Insulator, S = 1/2 2D Heisenberg Antiferromagnet

Hole-doped Case

Holes in a CuO_2 plane : Cu^{3+}



Same Symmetry for Cu^{2+} & Cu^{3+} => highly mobile holes



doped holes enter the O-*2p* orbitals and form Z-R singlets

***t-J*-model**: motion of holes in AF background

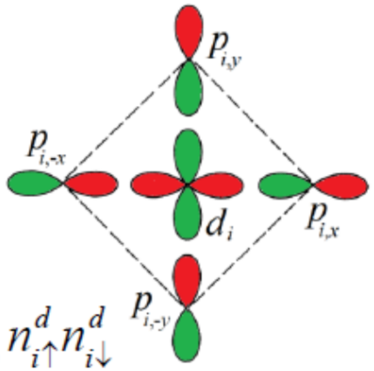
$$H = -t \sum_{\langle i,j \rangle, s} (c_{is}^{+} c_{js} + hc.) + J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

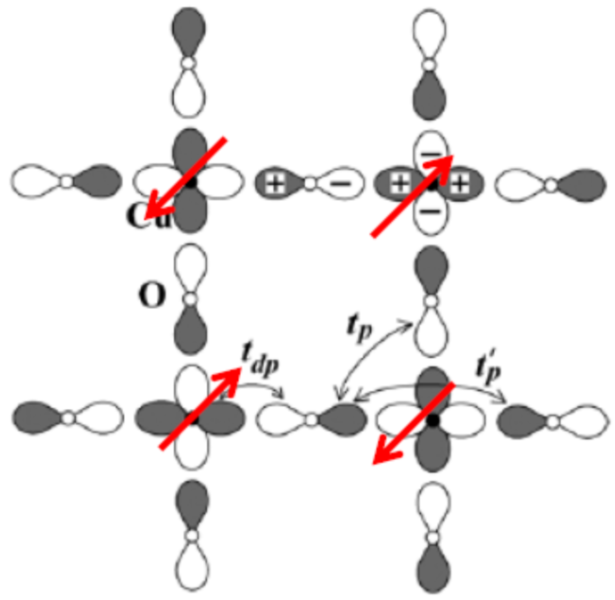
3 band model → 1 band model

3-band Hubbard model

Minimal 3-band model

Cu: $3d_{x2\text{-}y2}$(d), Planar O: $2p_x$,$2p_y$

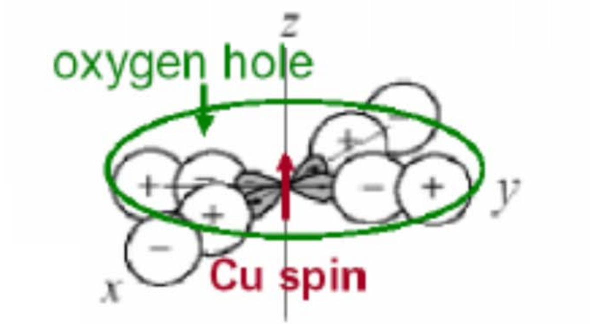




$$H=\varepsilon_p\sum_{i,\alpha,\sigma}p^\dagger_{i,\alpha,\sigma}p_{i,\alpha,\sigma}+\varepsilon^0_d\sum_{i,\sigma}d^\dagger_{i,\sigma}d_{i,\sigma}+U\sum_i n^d_{i\uparrow}n^d_{i\downarrow}$$
$$-t_{pd}\sum_{i,\sigma}\left[d^\dagger_{i,\sigma}\left(p_{i,x,\sigma}-p_{i,y,\sigma}-p_{i,-x,\sigma}+p_{i,-y,\sigma}\right)+\text{H.c.}\right]$$
$$-t_{pp}\sum_{i,\sigma}\left[p^\dagger_{i,x,\sigma}\left(p_{i,-y,\sigma}-p_{i,y,\sigma}\right)+p^\dagger_{i,-x,\sigma}\left(p_{i,y,\sigma}-p_{i,-y,\sigma}\right)+\text{H.c.}\right]$$

Example set: t_{pd} = 1 eV, t_{pp} = 0.5 eV, ε_p-ε_d^0= 6.5 eV, U = 10 eV

Zhang-Rice singlet (ZRS)

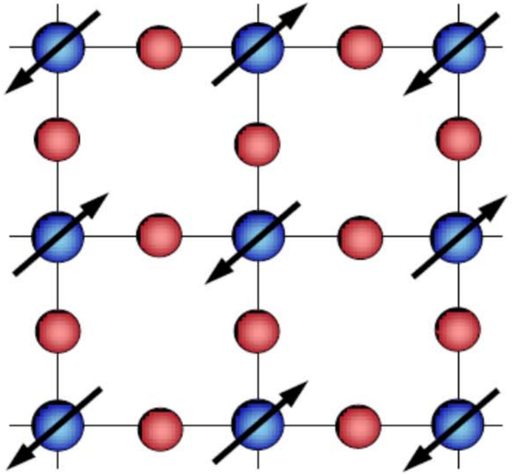


This is an Anderson Lattice Model:

- ❑ Charge transfer insulator when undoped (one hole
- ❑ Charge fluctuations quenched by large-U on Cu: onl
  → local moment (S=1/2) on Cu. Large quantu
- ❑ Doped holes go to oxygen; new states introduced ir

Simplest description of the cuprates

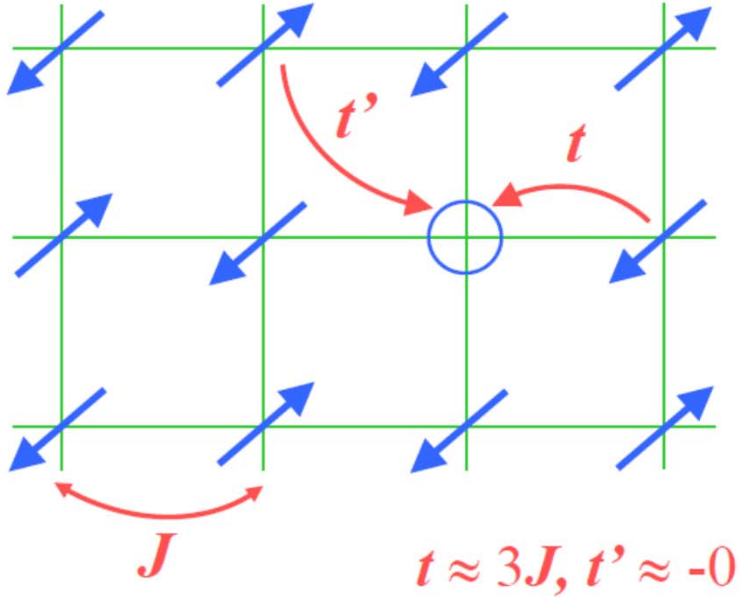
Undoped CuO_2 plane
AF Heisenberg model



Dope holes

$\Rightarrow$

Doped CuO_2 plane
One-band t-J model



$$H = J\sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

No charge fluctuations

$$H = -t\sum_{i\neq j} P_G\left(c_{i\sigma}^{\dagger}c_{j\sigma} + \mathrm{H.c.}\right)P_G + J\sum_{\langle i,j\rangle}\left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4}n_i n_j\right)$$

P_G: projection operator
No double occupation

Connection to Hubbard model in the large-U limit

3. Plain Vanilla Version of RVB Theory

Examples in solving quantum many-body problems

(1) BCS theory for SC (1956) **---- BCS wavefunction (Schrieffer Ansatz)** **---- Pairing Hamiltonian**
(2) Laughlin's theory for fractional QHE (1983) **--- Laughlin's wavefunction (Laughlin Ansatz)**

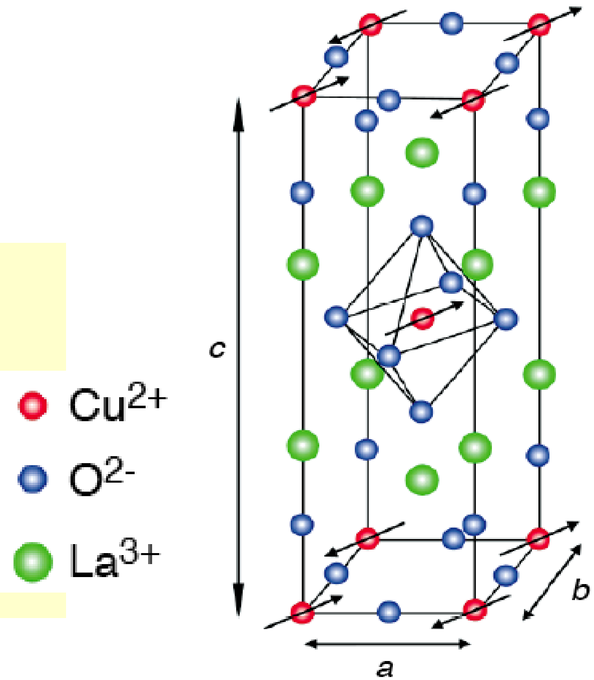
Anderson's RVB proposal for La_2CuO_4

PW Anderson, Science **235**, 1196 (1987)

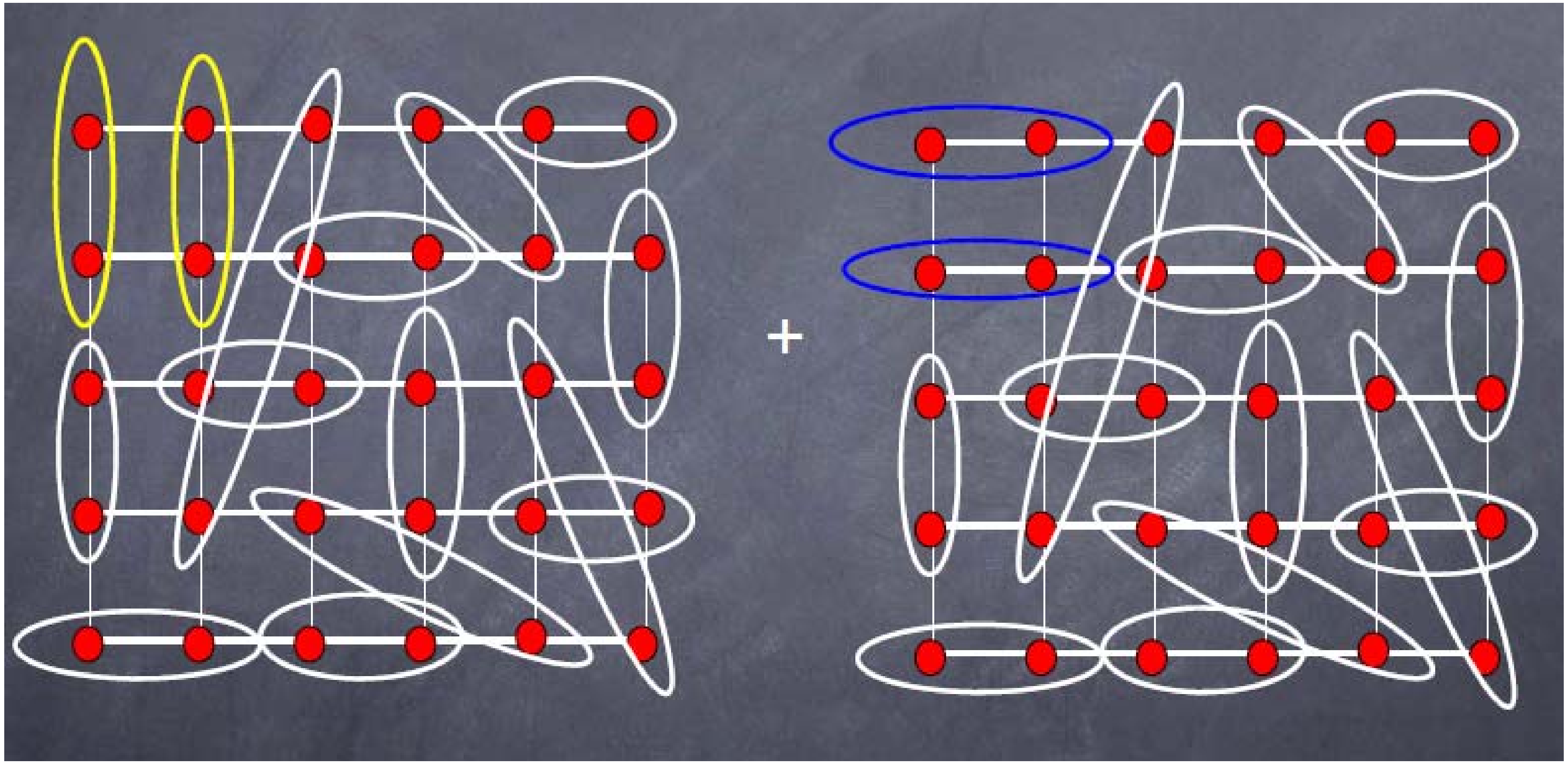
"The oxide superconductors, particularly those … base on La_2CuO_4, … tend … to occur near a metal-insulator transition … . This insulating phase is proposed to be the long-sought 'resonating-valence-bond' state or 'quantum spin liquid' hypothesized in 1973. This insulating magnetic phase is favored by low spin, low dimensionality, and magnetic frustration."

PW Anderson, Mat. Res. Bull. **8**, 153 (1973)
"Resonating Valence Bonds: A New Kind of Insulator"

Proposal for S=1/2 on a triangular lattice

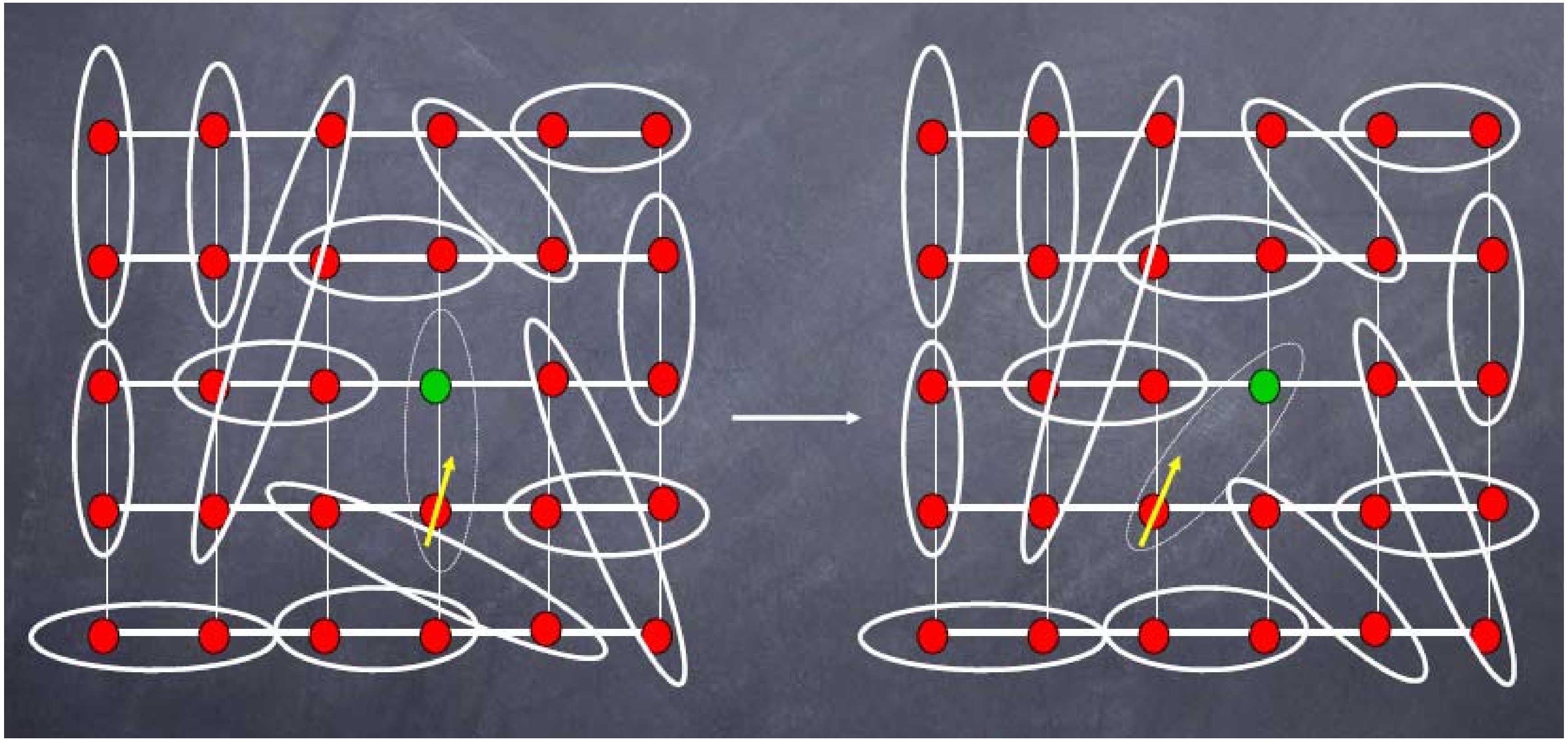


Resonant Valence Bond State



Anderson proposed an ansatz wavefunction for antiferromagnetic models: the Gutzwiller-projected BCS wavefunction, i.e., the RVB state (1987).

Spin and Charge Separation



Plain vanilla version of RVB theory

Minimal Hamiltonian: t-J model

$$H = -t \sum_{\langle i,j \rangle} c^{+}_{i,\sigma} c_{j,\sigma} + J \sum_{\langle i,j \rangle} \vec{S}_i \bullet \vec{S}_j$$

Constraint: 0,or 1 electron each site

2D RVB State which is a superposition of configurations
with Singlet Pairs can be written as a projected BCS state.

$$\left|\Psi_{RVB}\right\rangle = P\left|\Psi_{BCS}\right\rangle, P = \prod_i (1 - n_{i,\uparrow} n_{i,\downarrow})$$

$$\left|\Psi_{BCS}\right\rangle = \prod_k (u_k + v_k c^{+}_{k,\uparrow} c^{+}_{-k,\downarrow})\left|0\right\rangle$$

Explains many features of Hi-Tc: Anderson et al J Phys C '04

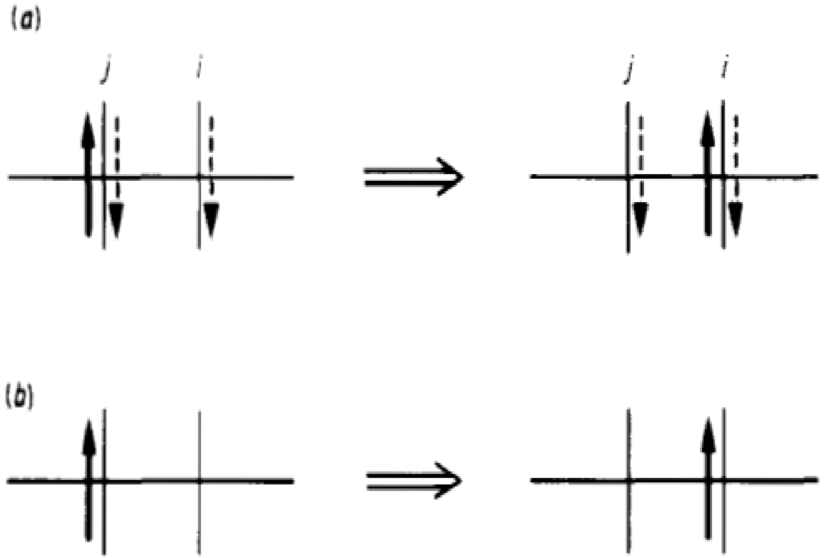
Renormalized Mean Field Theory

$$H_{t-J} \text{ on } |RVB\rangle = P|BCS\rangle \Rightarrow H_{eff} \text{ on } |BCS\rangle$$

$$H_{eff} = g_t T + g_s J \sum_{\langle i,j \rangle} \vec{S}_i \bullet \vec{S}_j$$

$$g_t \approx \frac{2x}{1+x}, g_s \approx \frac{4}{(1+x)^2}$$

x: hole doping concentration (statistical independence of population --- **Gutzwiller approximation**)



The possible hopping processes (a) in the **non-projected pairing state** and (b) in the **projected BCS state**. The spin with broken arrows are optional in the (a) configurations

Supercond. Sci. Technol. 1 (1988) 36–46. Printed in the UK

A renormalised Hamiltonian approach to a resonant valence bond wavefunction

F C Zhang, C Gros, T M Rice and H Shiba†
Theoretische Physik, ETH-Hönggerberg, CH 8093 Zurich, Switzerland

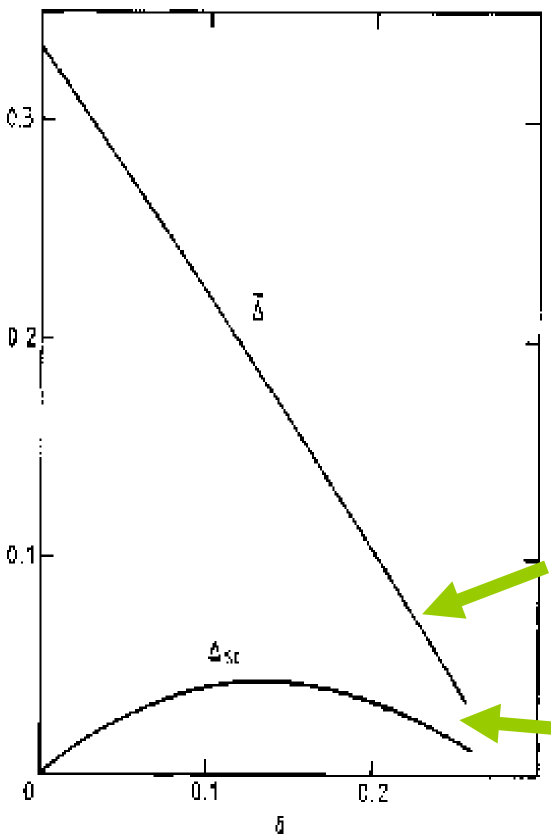
Received 1 March 1988, in final form 22 April 1988

Plain vanilla RVB (88')

(1). D-wave pairing: confirmed in expt. (1994)
(2). Pseudogap (large Q.P. energy) expt. reported 1990's
(3). Approx spin-charge separation (renormalized in different ways) expt.: small superfluid density Tc & Drude weight v.s doping
(4). Gutzwiller approximation works quite good, comparing with variational Monte Carlo results.

Pseudogap v.s. SC order (88')

Theory (1988)



$$\Psi_{RVB}^{q.p.} = P\Psi_{BCS}^{q.p.}$$

$$E_k = \sqrt{\xi_k^2 + (\tilde{\Delta}(\cos k_x - \cos k_y))^2}$$

$$\Delta_{SC} = g_t \tilde{\Delta} \approx 2\delta\tilde{\Delta}, g_t = \frac{2\delta}{1+\delta}$$

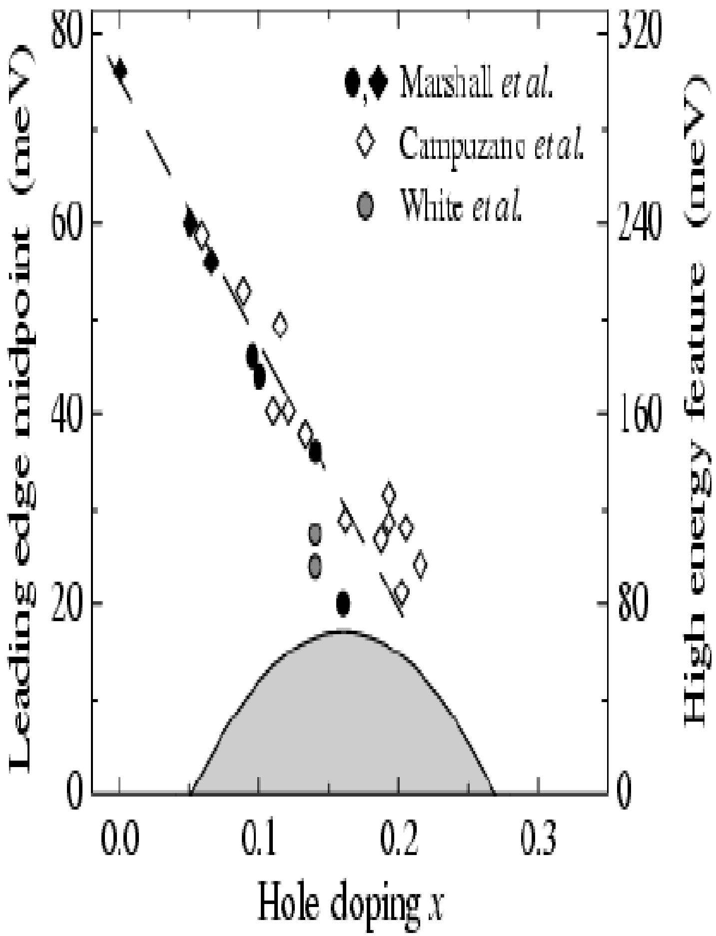
amplitude of excitation energy

SC order parameter

hole concentration

$t/J = 5$

Experiment



Why High Tc still controversial

1) Other competing phases

----charge density wave

----antiferromagnetism

----stripe phase

---- etc

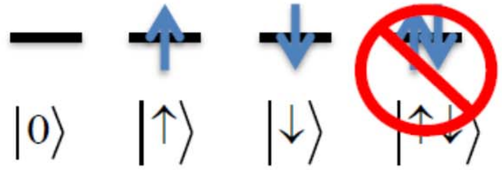
but the key is unusual SC

2) Numerical controversial details t-t'-J etc.

Slave boson formulation of the t-J model

$$H_{tJ} = -t\sum_{i\neq j}\left(c_{i\sigma}^{\dagger}c_{j\sigma} + \mathrm{H.c.}\right) + J\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j + \text{constraints}\ (c_{i\sigma}^{\dagger}c_{i\sigma}\leq 1)$$

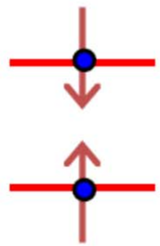
- Slave-boson for projected Hilbert space:



Spinless boson

$$c_{i\sigma}^{\dagger} = f_{i\sigma}^{\dagger}b_i$$

Spin-carrying fermion (spinon)



empty site: $|\text{empty}\rangle = b_i^{\dagger}|0\rangle$

singly occupied site: $f_{i\downarrow}^{\dagger}b_i|\text{empty}\rangle$

singly occupied site: $f_{i\uparrow}^{\dagger}b_i|\text{empty}\rangle$

Each site is and must be occupied by either a boson or a fermion

Constraints → equality: $f_{i\sigma}^{\dagger}f_{i\sigma} + b_i^{\dagger}b_i = 1$ completeness

Enforced by Lagrange multipliers

$$H = \sum_{i\neq j} t\left(b_j^{\dagger}b_i f_{i\sigma}^{\dagger}f_{j\sigma} + \mathrm{H.c.}\right) + J\sum_{\langle i,j\rangle}\mathbf{S}_i^f\cdot\mathbf{S}_j^f + \sum_i i\lambda_i\left(\sum_{\sigma} f_{i\sigma}^{\dagger}f_{i\sigma} + b_i^{\dagger}b_i - 1\right)$$

Slave boson mean field theory

RVB decoupling of exchange interaction:

Not in AFM

$$\chi_{ij} = \left\langle f^{\dagger}_{i\sigma} f_{j\sigma} \right\rangle \qquad \Delta_{ij} = \left\langle f_{i\downarrow} f_{j\uparrow} - f_{i\uparrow} f_{j\downarrow} \right\rangle \qquad b_i = \left\langle b_i^{\dagger} \right\rangle = \left\langle b_i \right\rangle$$

Paramagnetic valence bond

Spin-singlet pairing

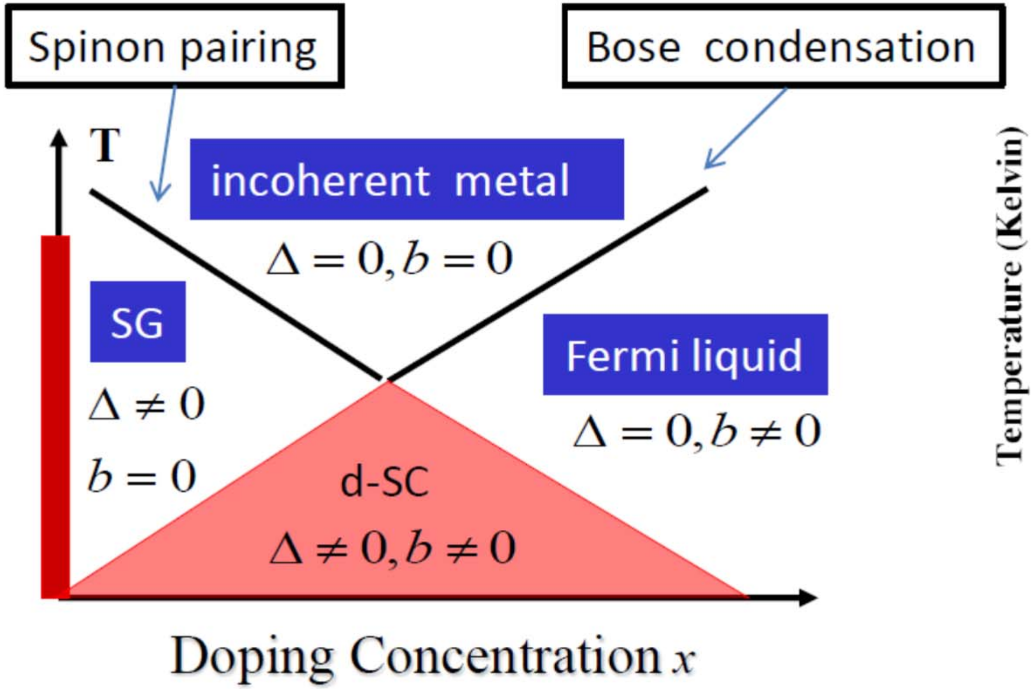
$$S_i^f \cdot S_j^f = -\frac{1}{4}[\chi_{ij}^{*} f^{\dagger}_{i\sigma} f_{j\sigma} + \Delta_{ij}^{*}(f_{i\downarrow} f_{j\uparrow} - f_{i\uparrow} f_{j\downarrow}) + \mathrm{h.c.}] + \frac{1}{4}(\left|\chi_{ij}\right|^2 + \left|\Delta_{ij}\right|^2).$$

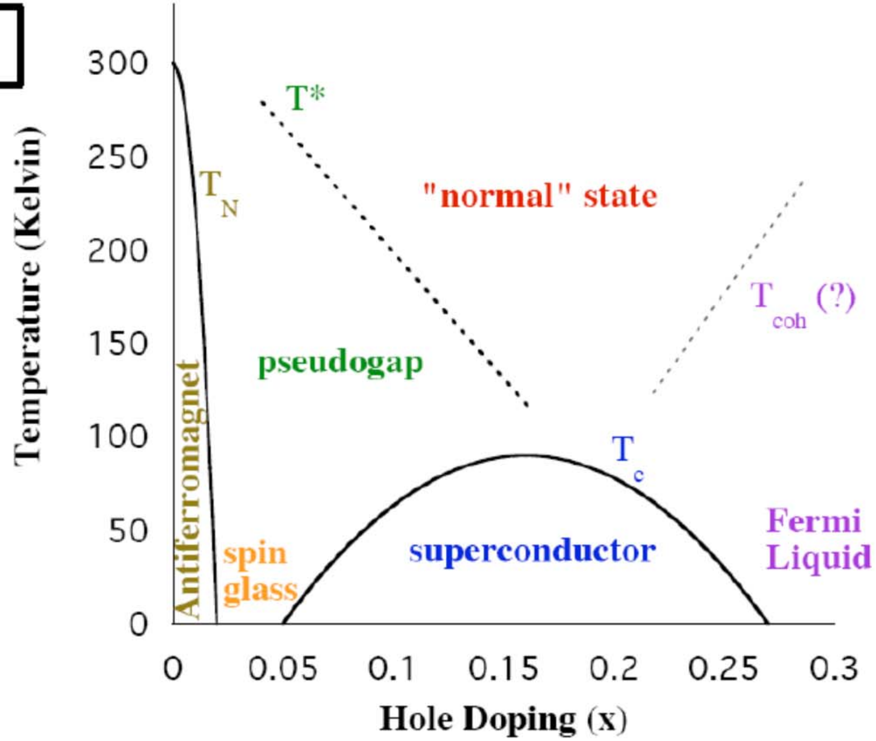
Uniform mean-field solution:

$$\chi_{ij} = \chi \neq 0 \qquad \Delta_{ij} = \Delta \tau_{ij} \qquad b_i = b \qquad \lambda_i = \lambda$$

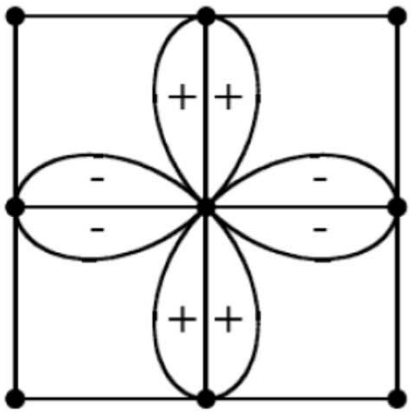
τ_{ij} – symmetry of the pair: s-wave, d-wave, ...

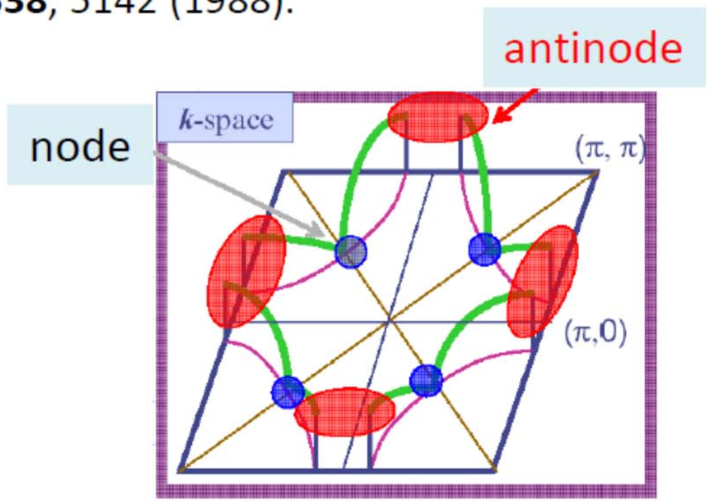
Uniform mean-field phase diagram





Kotliar and Liu, PRB**38**, 5142 (1988).

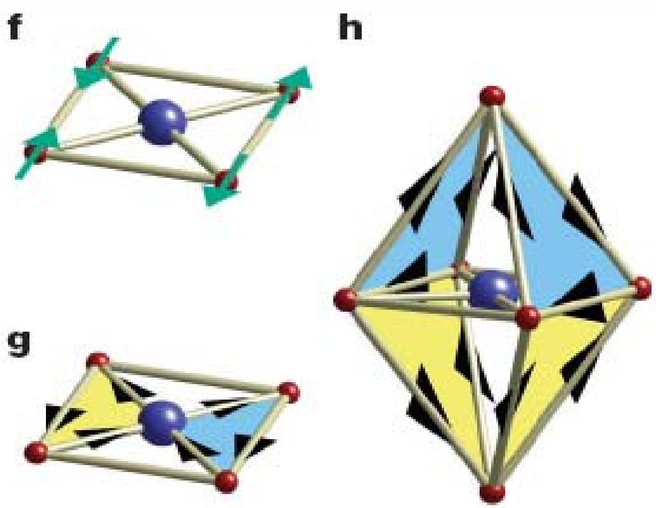




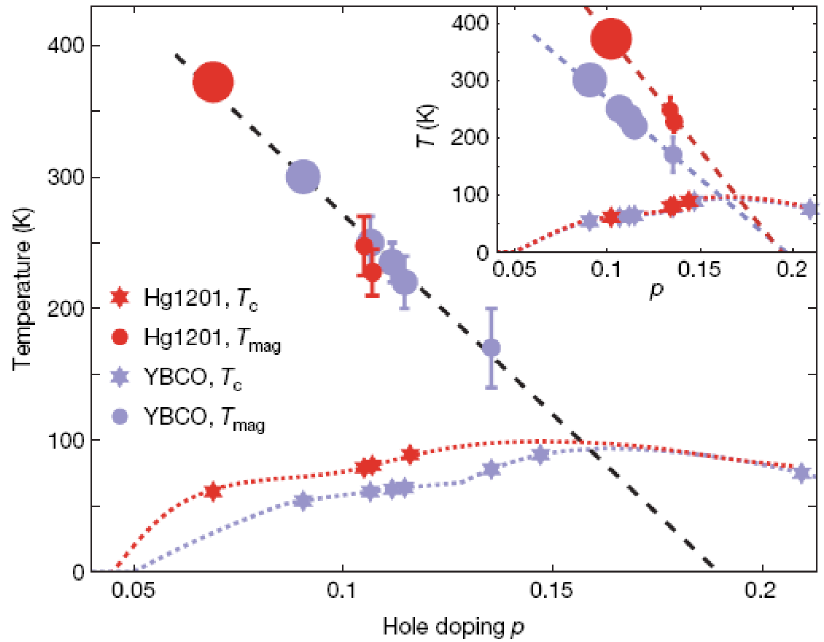
- Superexchange → d-wave SC
- PG: spin pairing gap
- T_c is set by phase coherence below optimal doping

Orbital Current State: 3-band Hubbard Model

Theoretically, C. Varma believes that 3 band Hubbard model with interaction V between Cu and O is needed. He proposes the existence of orbital currents in the plane between Cu and O. These currents occur within the unit cell.



Orbital currents have been observed by neutron scattering. The onset of these currents seem related to T*, the pseudo-gap scale. There is also reports of T breaking (ferromagnetic like) by polar Kerr effect at slightly lower temperature.



Li ..Bourges, Greven, Nature 455, 372 (2008).

4. Numerical Studies on t-J model

Difficulty of Quantum Many Body Physics

- Physical properties for a N-site lattice ?

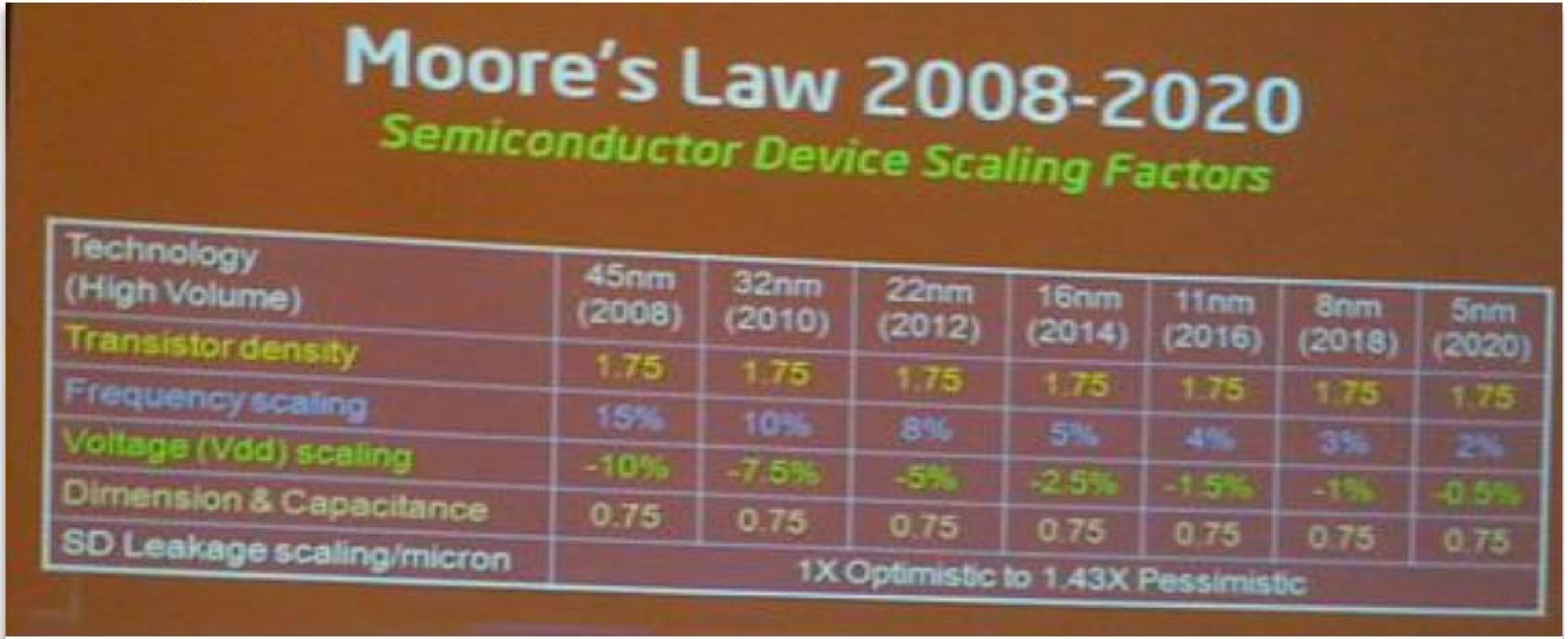
$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \ , \qquad H\psi = E\psi \ , \qquad Z = Tr\,\exp(-\beta H)$$

- Exact analytical results for special cases only (for instance, 1D Hubbard model, Tomonaga-Luttinger model).

- Hilbert space dimension $= 4^N$
  $\Rightarrow$ Exact diagonalizations up to $N \approx 16$ only.

- "Exact" numerical methods for $N \gg 1$?

Rapid Progress of Computational Capacity



Moore's Law 2008-2020

Semiconductor Device Scaling Factors

Technology (High Volume)	45nm (2008)	32nm (2010)	22nm (2012)	16nm (2014)	11nm (2016)	8nm (2018)	5nm (2020)
Transistor density	1.75	1.75	1.75	1.75	1.75	1.75	1.75
Frequency scaling	15%	10%	8%	5%	4%	3%	2%
Voltage (Vdd) scaling	-10%	-7.5%	-5%	-2.5%	-1.5%	-1%	-0.5%
Dimension & Capacitance	0.75	0.75	0.75	0.75	0.75	0.75	0.75
SD Leakage scaling/micron	1X Optimistic to 1.43X Pessimistic						

1000 fold increase in performance in 10 years:

> previously: double transistor density every 18 months = 100X in 10 years
frequency increased

> now: "only" 1.75X transistor density every 2 years = 16X in 10 years
frequency almost the same

Need to make up a factor 60 somewhere else

Source: Rajeeb Hazra's (HPC@Intel) talk at SOS14, March 2010

Numerical Methods for Lattice Fermions

Numerical Exact Diagonalization (ED)
High Temperature Expansion
Density Matrix Renormalization Group (DMRG)
Variational Monte Carlo
Functional RG
Dynamical Mean Field Theory (DMFT)

RPA, Linked Cluster Expansion
Quantum Monte Carlo (QMC)
    auxiliary field
    worldline
Tensor Network States Algorithms
  PEPS, MERA

……

- **Small clusters**
- **High to moderate T**
- **1D or ladder system (王孝群）**
- **Trial wave functions (王强华）**
- **Small Cluster+DMFT （李建新）**
- **Negative sign**

Exact Diagonalization

Main idea

- Solving the Schrodinger equation numerically
- Sparse matrix, but for quantum many body systems the vector space dimension grows exponentially! For a Hubbard model, $D \sim 4^L$; Heisenberg Model, $D \sim 2^L$, …

(**Dimension reduction by imposing symmetry**)

The limits of ED (up to 2010)

● t-J models:

32 sites checkerboard with 2 holes

32 sites square lattice with 4 holes

up to 2.8 billion basis states

● Hubbard models

20 sites square lattice at half filling, 21 sites triangular lattice

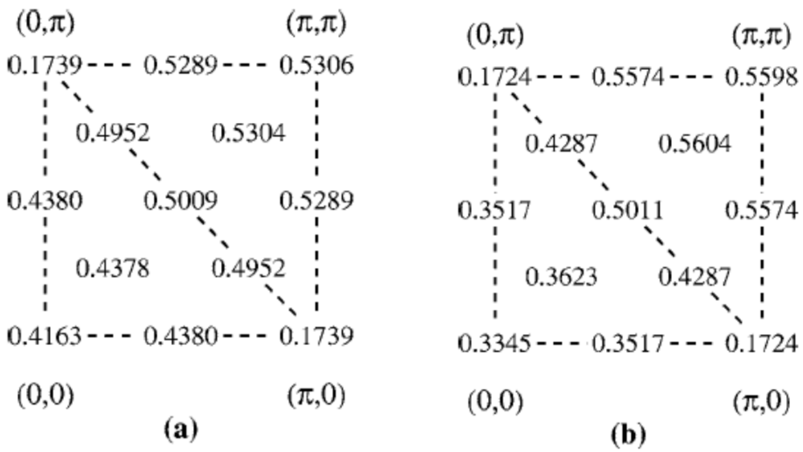
up to 160 billion basis states

ED Study of t-J model

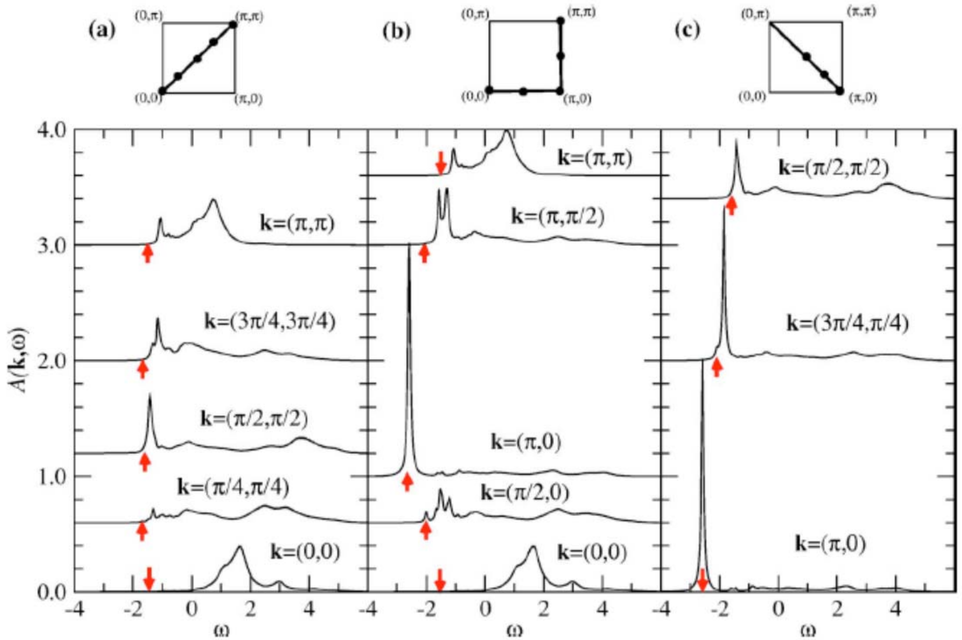
32-site cluster with 1,2,4 holes

TABLE I. Ground-state energies, momenta, and point group symmetries of the electron-doped model with N_c charge carriers. N_B is the number of bases in that particular subspace. The ground-state energy at half filling E_0^0 is −11.329 720.

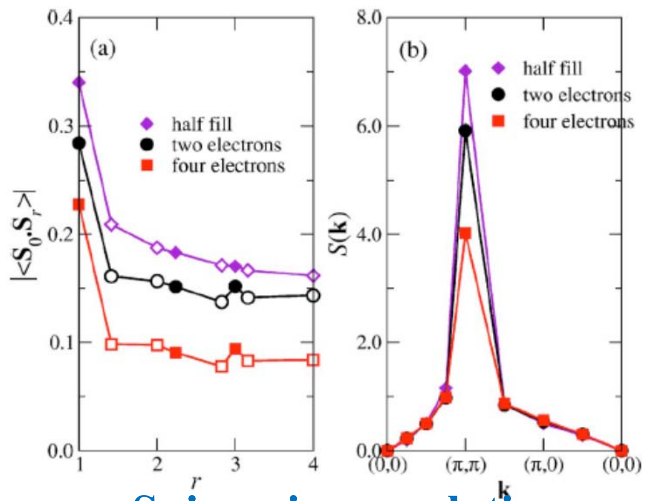
N_c	N_B	$E_0^{N_c}$	**k**	Symmetry
1	150 297 603	−13.913 616	$(\pi,0)$, $(0,\pi)$	
2	150 295 402	−16.601 689	(0,0)	$d_{x^2-y^2}$
4	2 817 694 064	−20.461 647	(0,0)	s



n(k) for 2 (a) and 4 (b) holes



Spectral function in k-space



Spin-spin correlations

Density Matrix Renormalization Group

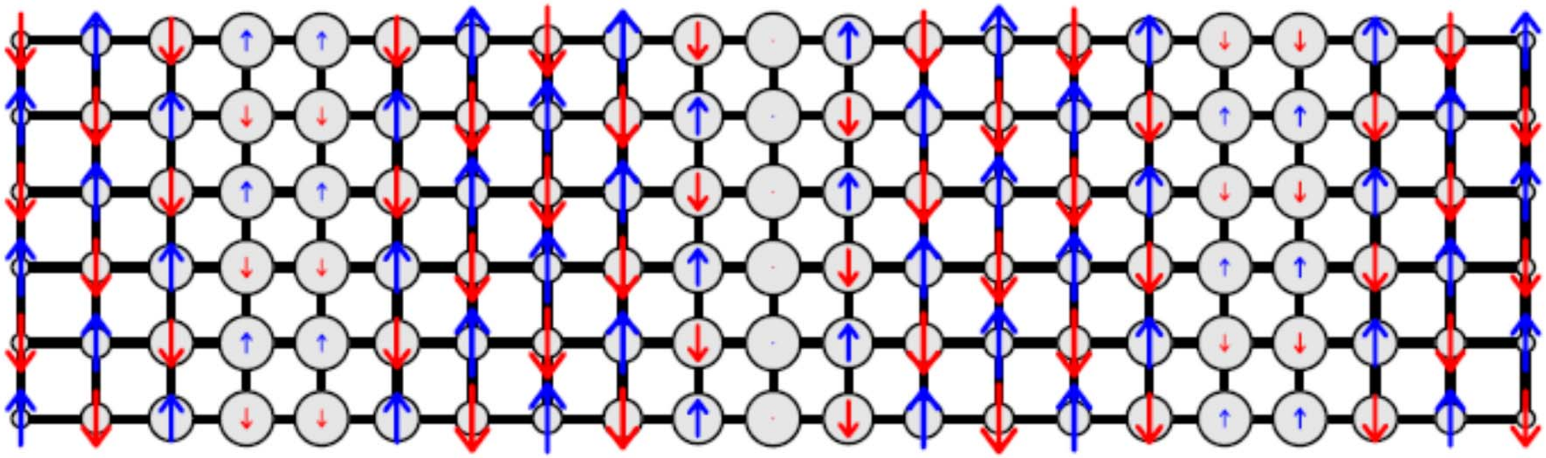
- DMRG Method rooted in Wilson's original RG procedure, whereby we systematically add degrees of freedom (sites or levels) until all have been treated.
- DMRG is a numerical technique for finding accurate approximations of the ground state and the low-energy excited states of strongly interacting quantum systems.
- Its accuracy is remarkable for one-dimensional systems with very little amount of computational effort. It is however limited by the dimensionality or range of interactions.

Advantage:

- For large systems
- Accuracy comparable to exact results
- Variational and non-Perturbative
- No problems with frustration or fermions

DMRG Study of t-J model

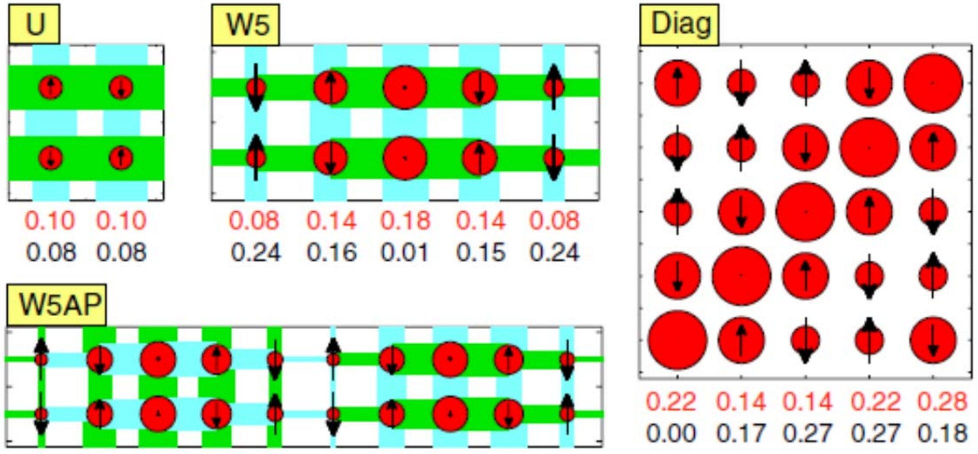
Stripe phase



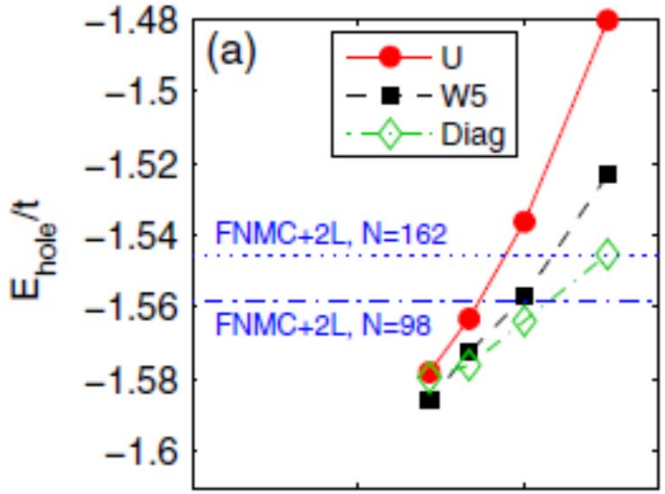
21×6 **ladder with 12 holes**

No spontaneous symmetry breaking
& emergence of superconducting order

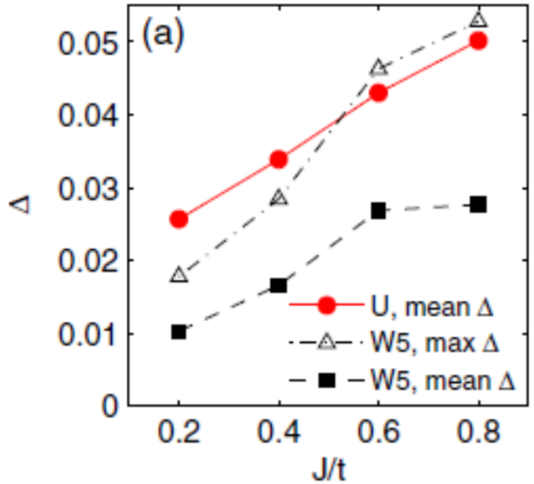
iPEPS Study of t-J model



Competing low energy states



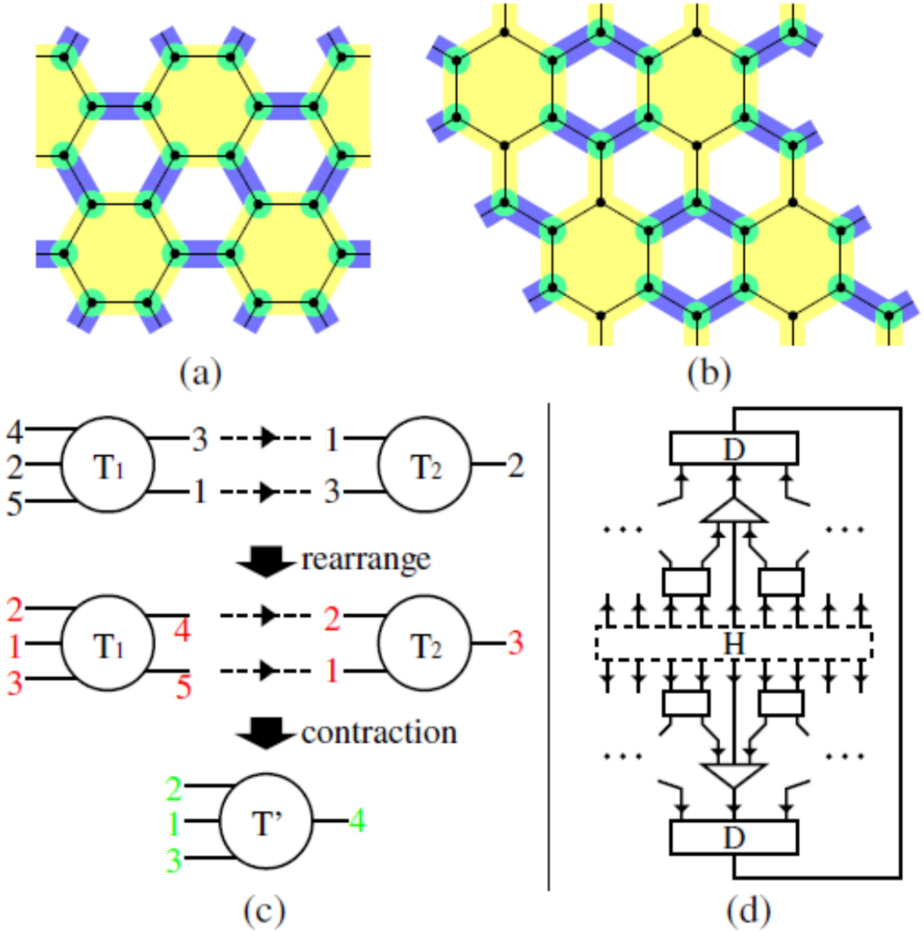
GS energies for competing states



SC gap versus J/t

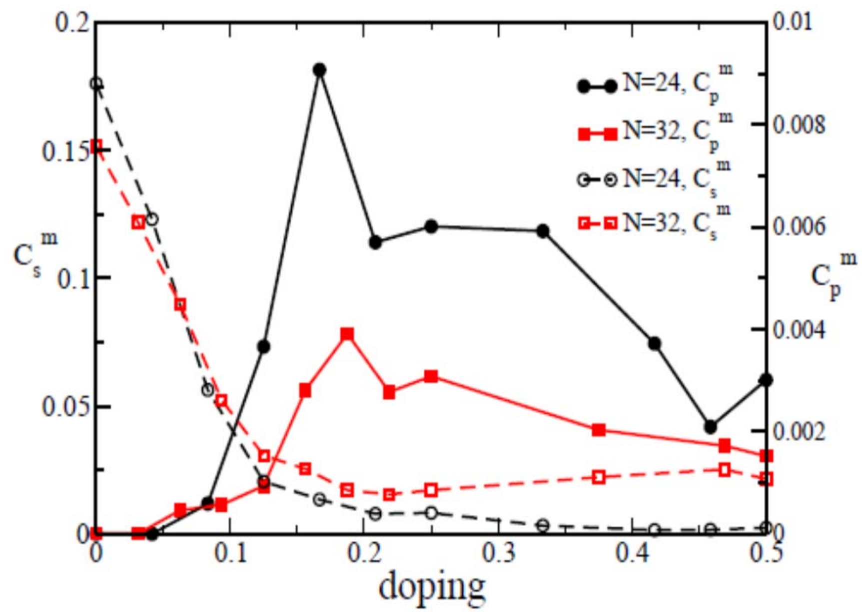
close competition between a uniform
d-wave superconducting state
and different stripe states.

Grassmann-Multiscale Entanglement Renormalization Ansatz (GMERA)



Tensor Network Structure and Optimization

Benchmark Calculation of t-J model



New algorithm aiming to find spontaneous symmetry breaking & emergence of superconducting order in larger system size

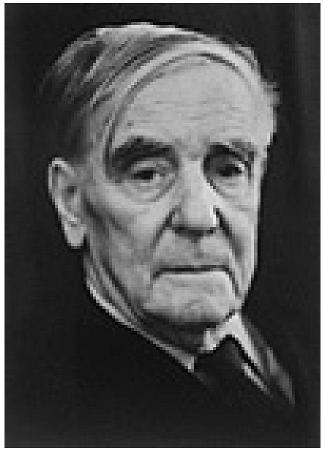
超流性/BEC/超冷原子

1. Background

2. Ultracold atoms in Optical Lattices

3. BEC-BCS Crossover

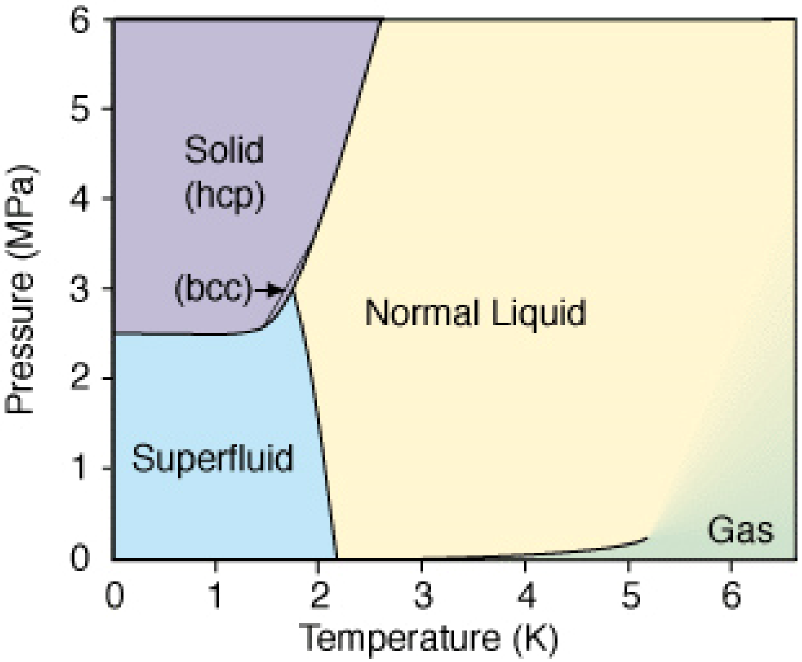
Superfluid *T ~ 4 K*



Pyotr L. Kapitsa 1938 (1978) Lev Landau 1941 (1962)

1938 Kapitsa discovered the superfluidity of ^{4}He --- first realization of Bose-Einstein Condensation

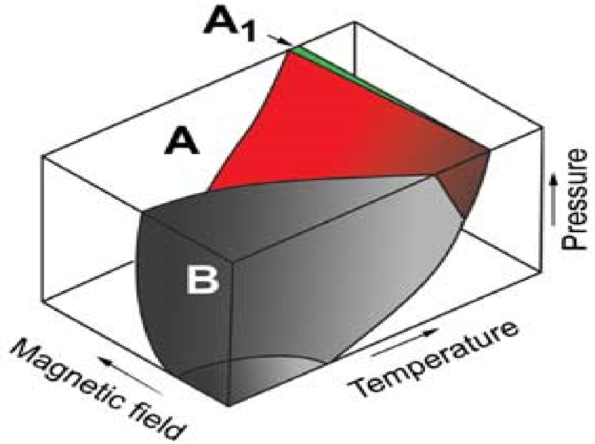
1940s Landau formulated the theory of ^{4}He superfluidity



Fermion Superfluid *$T \sim 10^{-3}$ K*

Early 1970s ^{3}He superfluidity was discovered

1996, 2003 Nobel prizes



自旋—轨道自发对称破缺

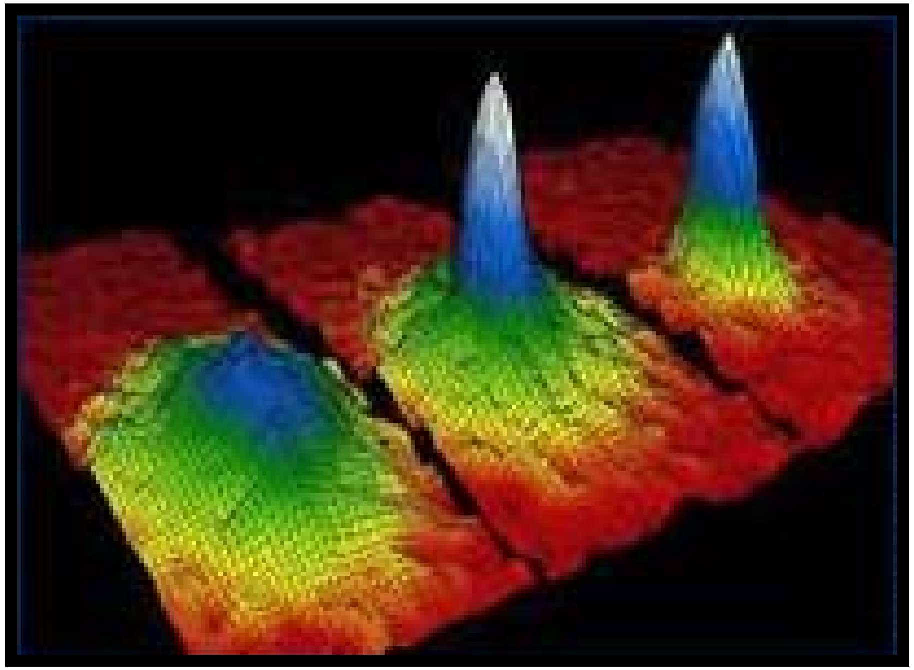
David M. Lee　D. D. Osheroff　R. C.Richardson　　Anthony Leggett

Bose-Einstein Condensation of diluted cold atoms

A technical breakthrough, stimulate the unification of quantum optics and condensed matter physics

Eric A. Cornell | Wolfgang Ketterle | Carl E. Wieman

2001 Nobel Prize



Velocity distribution function of ultracold ^{87}Rb atoms

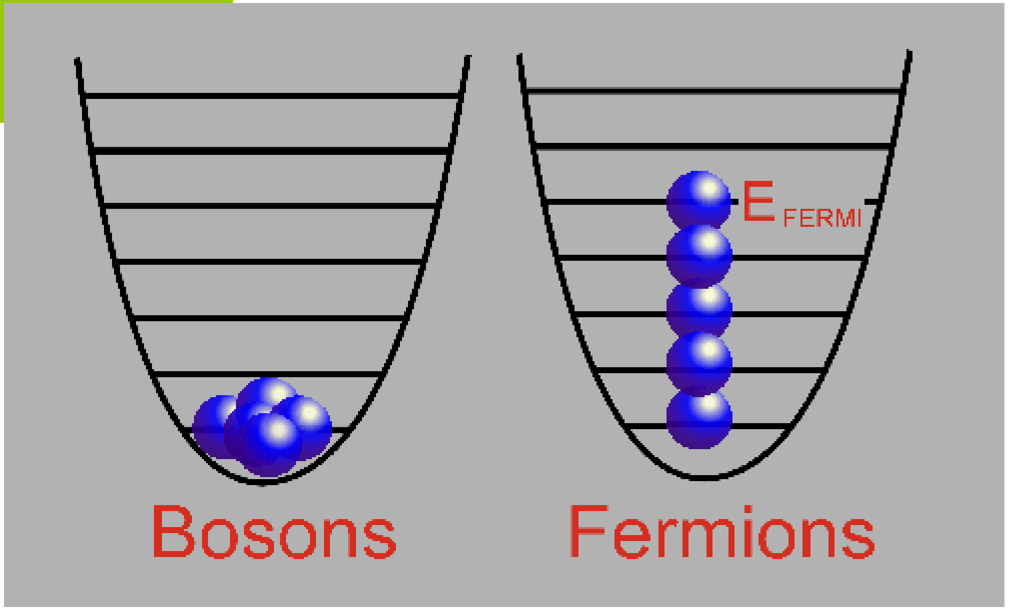
M. H. Anderson et al, *Science* 269, 198 (1995)

What is Bose-Einstein condensation ?

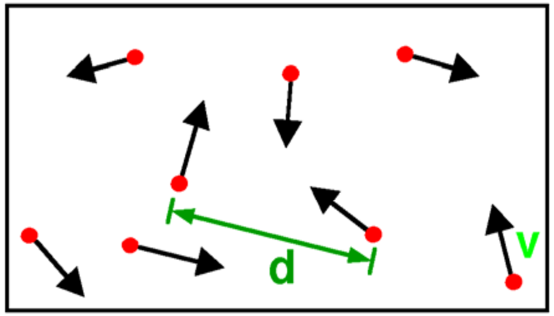
$\Psi(x_1, x_2) = \pm\Psi(x_2, x_1)$, + for boson and - for fermion

Therefore, for fermion we have $\Psi(x, x) = 0$,
i.e. fermions like to be far away,
but bosons do like to be close !

When T is small enough, noninteracting bosons like to stay in the lowest energy state, i.e. BEC



What is Bose-Einstein condensation (BEC)?

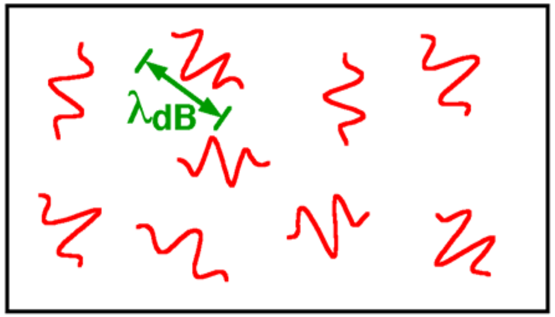


High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

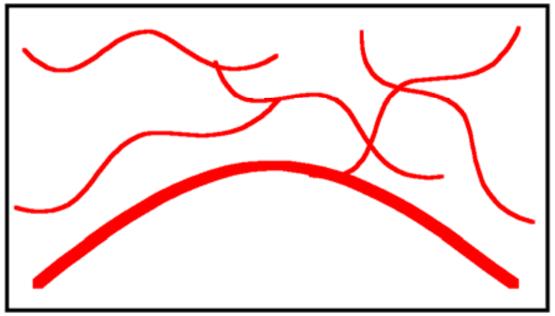


Low Temperature T:

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

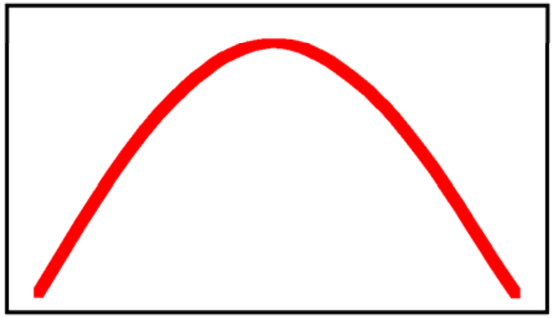
"Wave packets"



$T=T_{crit}$: Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

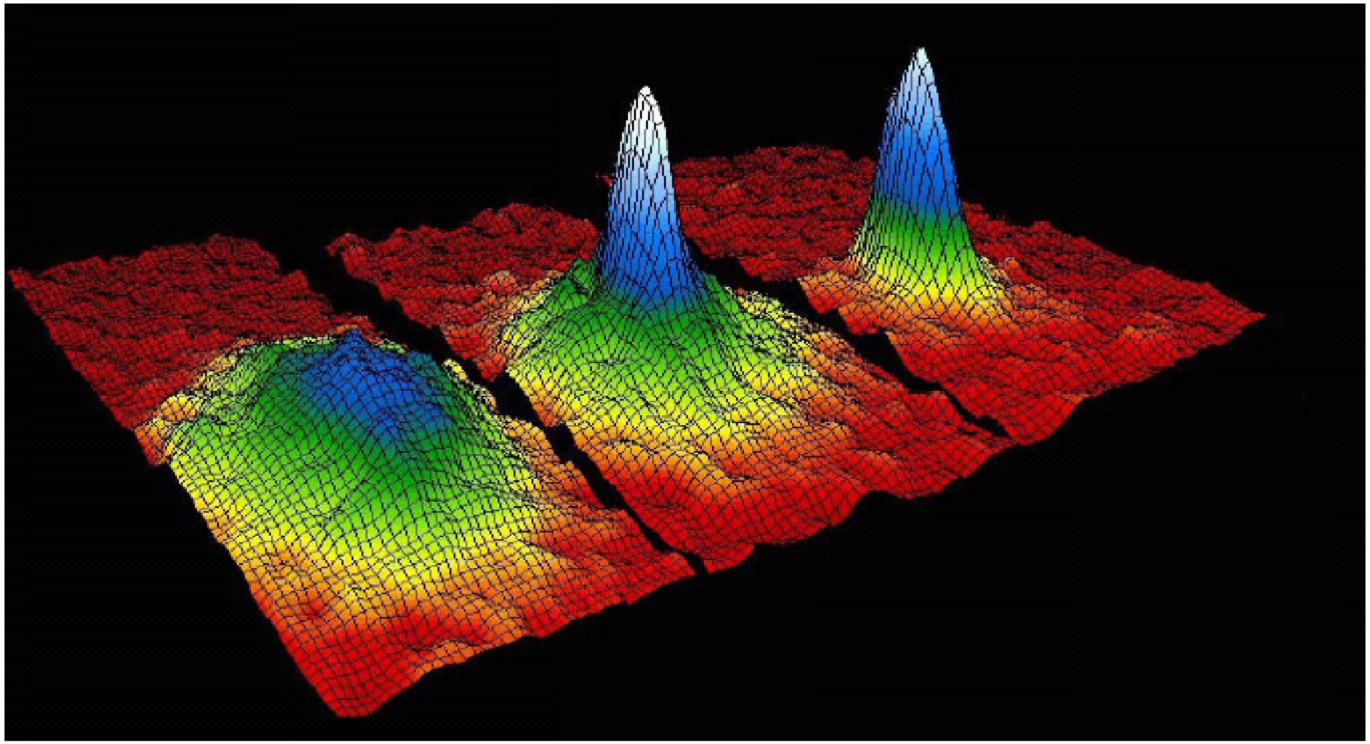


T=0: Pure Bose condensate

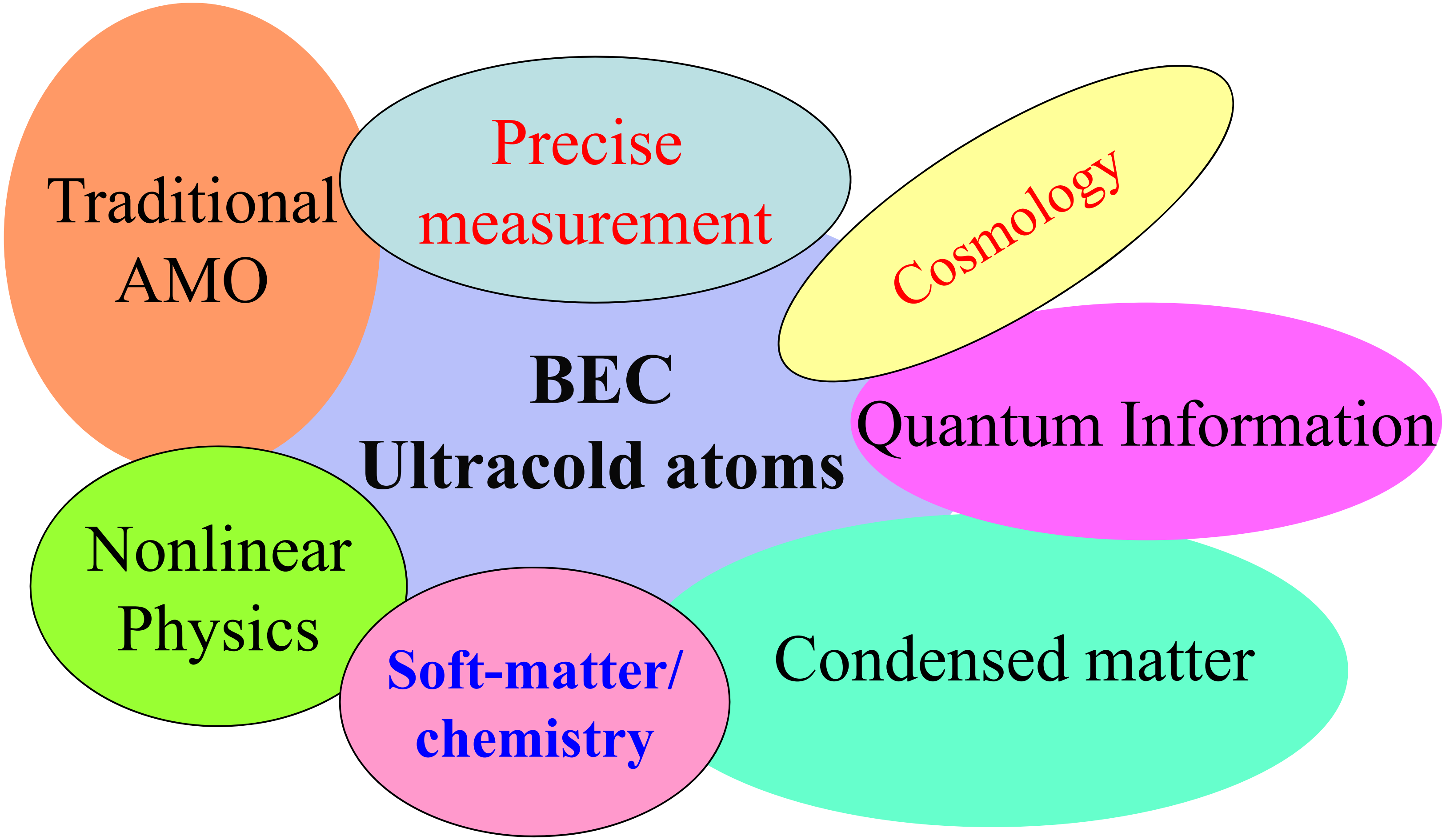
"Giant matter wave"

1. Background

Ultracold Atoms, BEC, Cooling techniques, Optical Lattice, Feshbach resonance



Ultracold Atoms



BEC can be widely applied in many branches of modern physics, condensed matter, nuclear physics, astrophysics, and atomic physics.

History of Bose-Einstein condensation (mainly exp.)

- Theoretical prediction
  1924/25 Bose and Einstein
- Superfluidity in liquid helium
  1938 Fritz London
  1983 Reppy et al. (Cornell): BEC of helium in vycor
- Excitons (complicated interactions - no BEC observed)
- Dilute atomic gases

Spin-polarized hydrogen:

agenda & experimental techniques (since late '70s)
MIT *(Greytak, Kleppner)* BEC '98
Amsterdam *(Silvera, Walraven)*
also: Harvard, BC, Turku, Cornell, Moscow

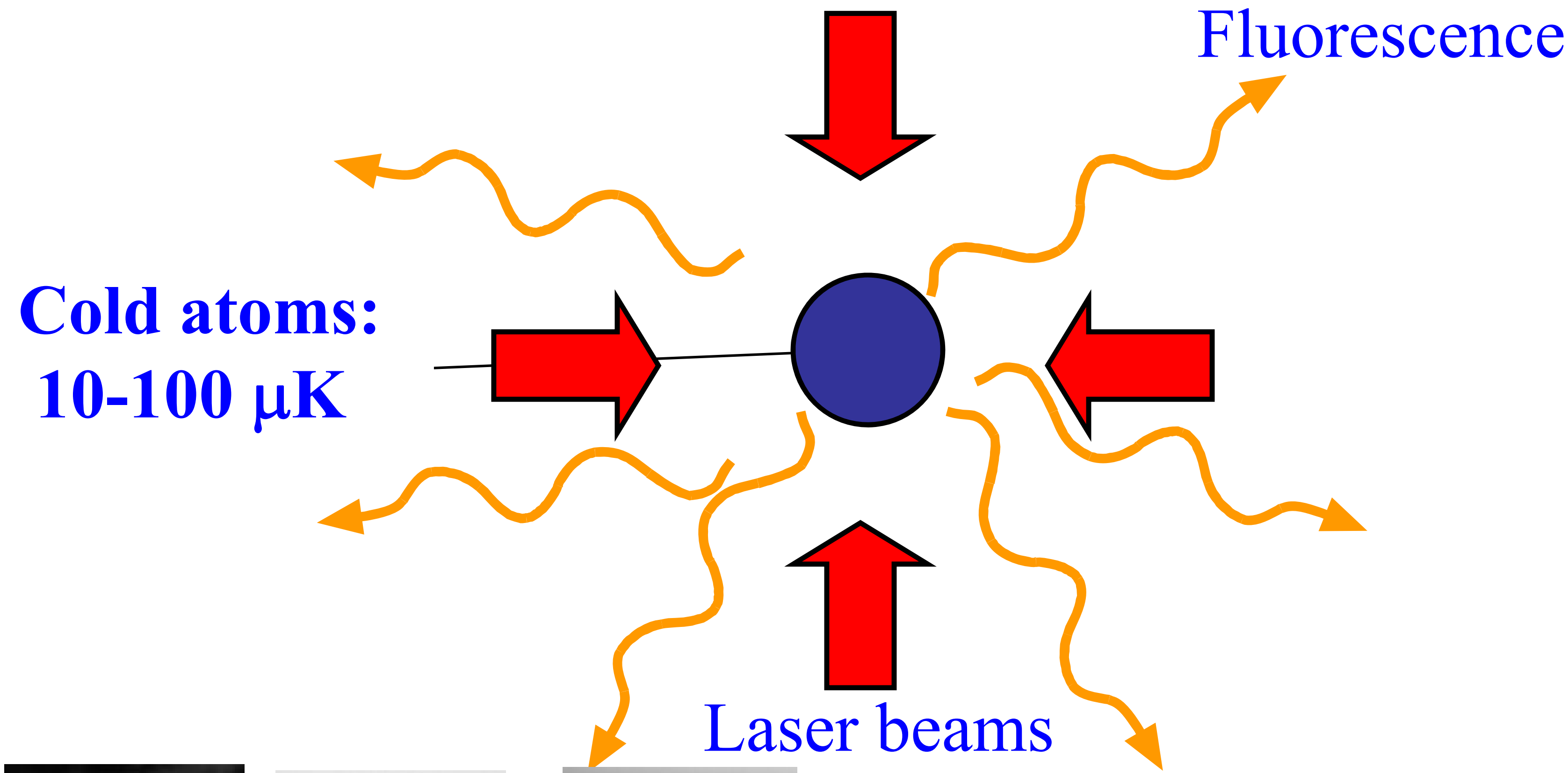
2D quantum degeneracy '98

Alkali atoms:

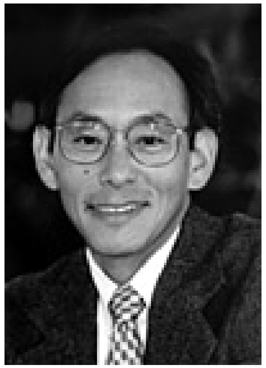
Laser cooling ('80s)
Focused programs in Boulder and at MIT (since early '90s)
June '95: Boulder *(Cornell/Wieman)*
Sept. '95: MIT *(W.K.)*
July '95 [indirect evidence]: Rice *(Hulet)*

now:
many experiments

Two Step Cooling techniques (I)



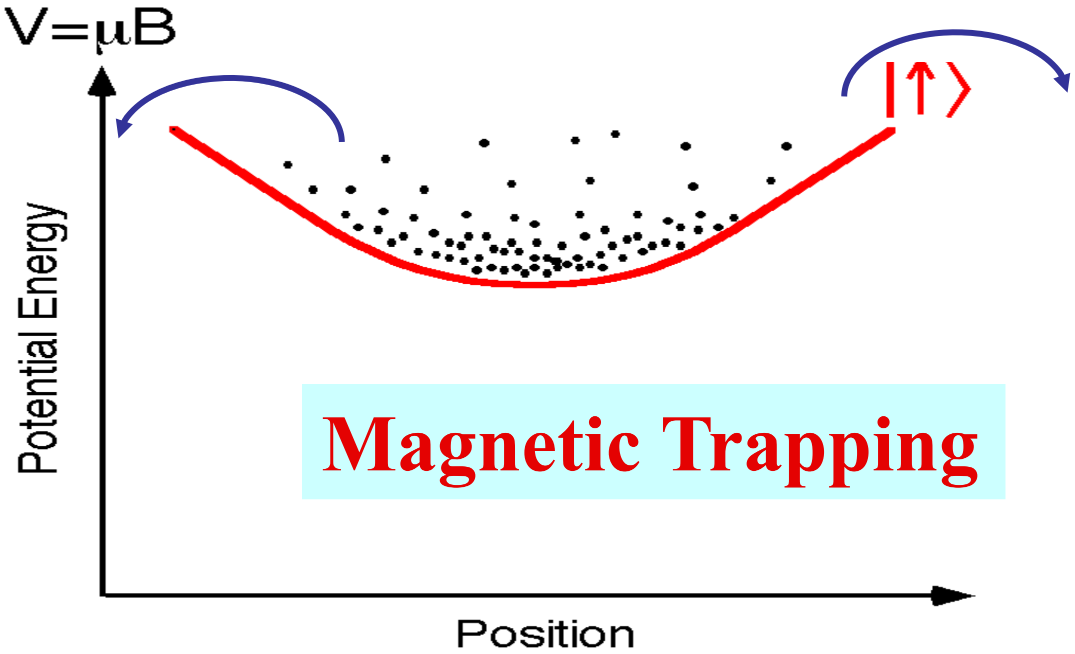
→Laser cooling



朱棣文 Cohen-Tannoundji Phillips

1997 Nobel Prize

Two Step Cooling techniques (II)



Magnetic Trapping

Cold atoms:
10 – 100 nK

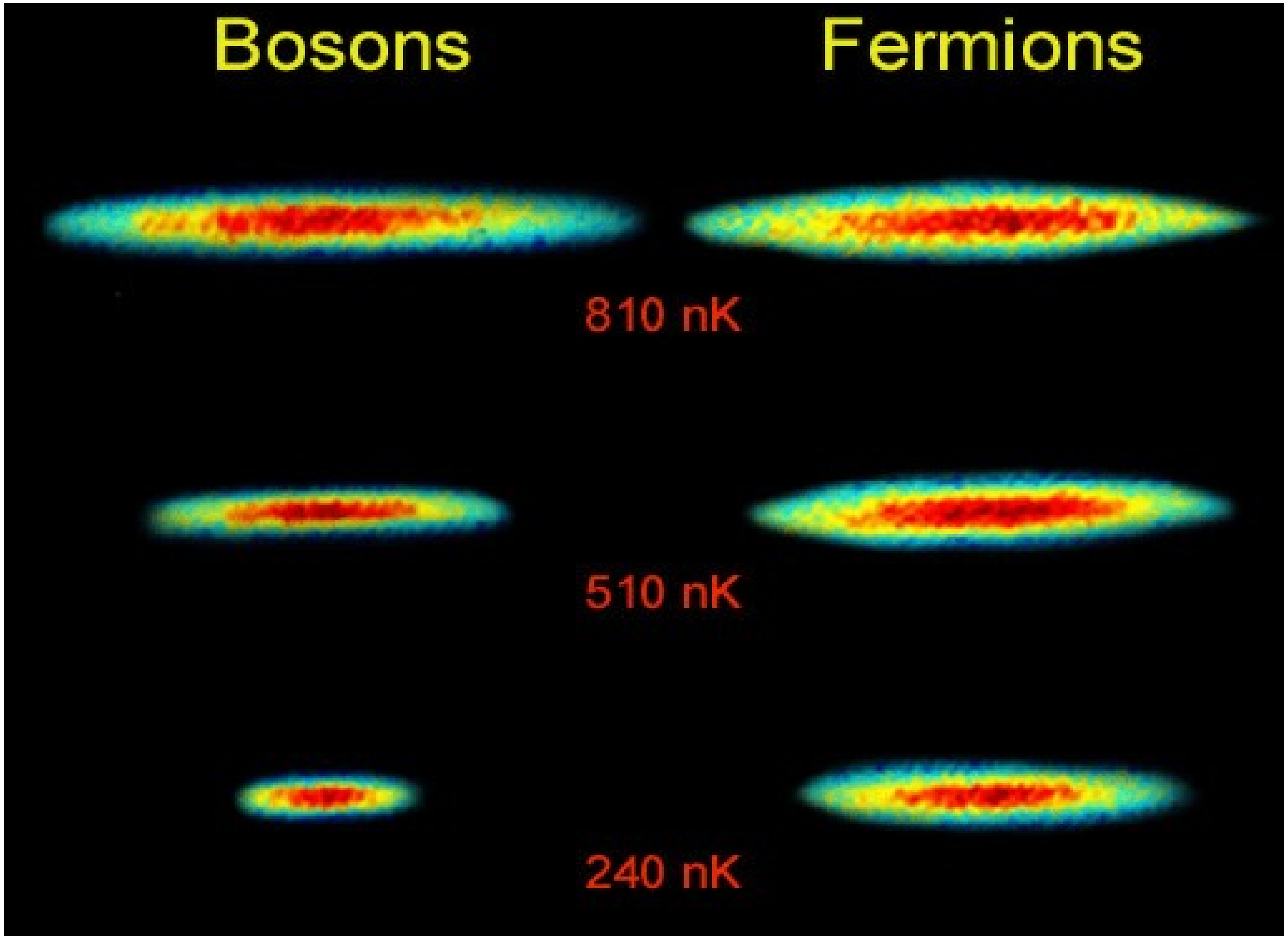
→Evaporative Cooling

Observing Statistics

High T:
Boltzmann
distribution

Hulet (2001)

Low T:
Degenerate gas



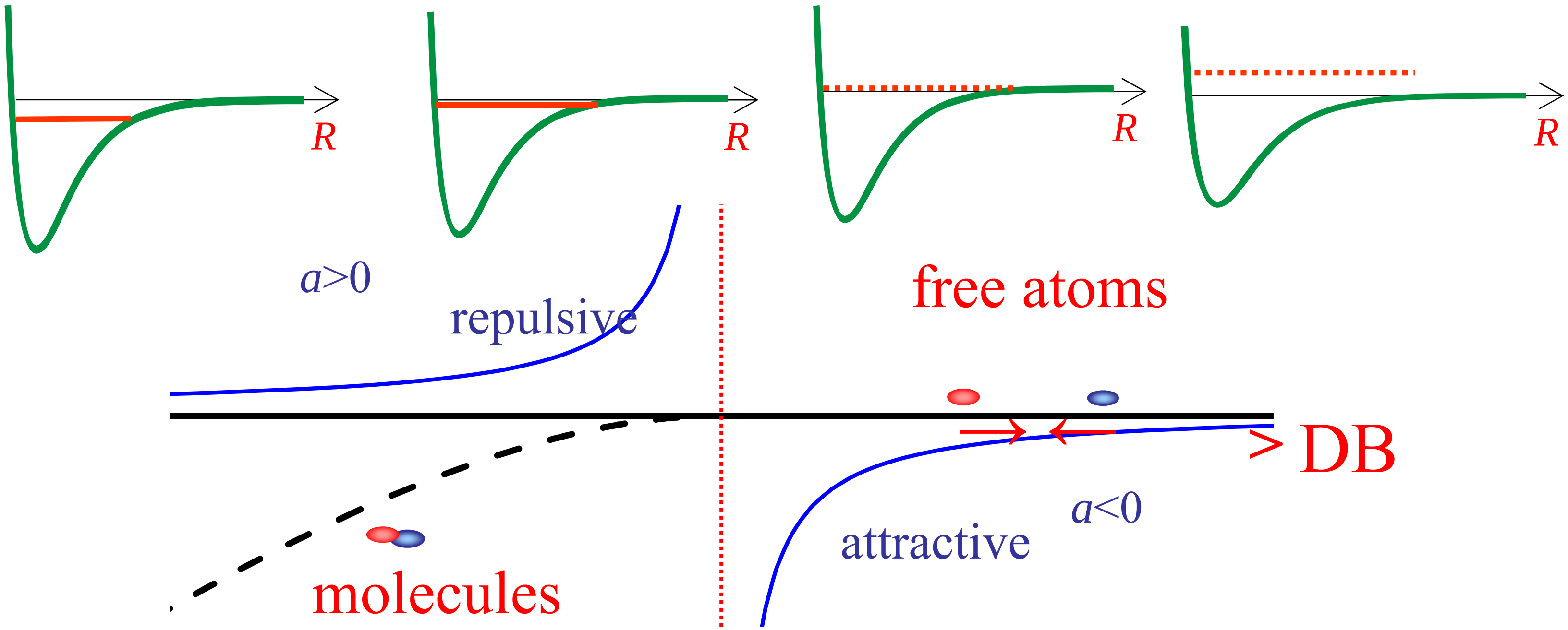
Feshbach Resonances

Feshbach Resonance: Controlling Interactions:

- Interactions are characterized by the s-wave scattering length, a

  $a > 0$ repulsive, $a < 0$ attractive

  Large $|a| \rightarrow$ strong interactions
- In an ultracold atomic gas, bound state can be controlled by an external magnetic field →we can control a!

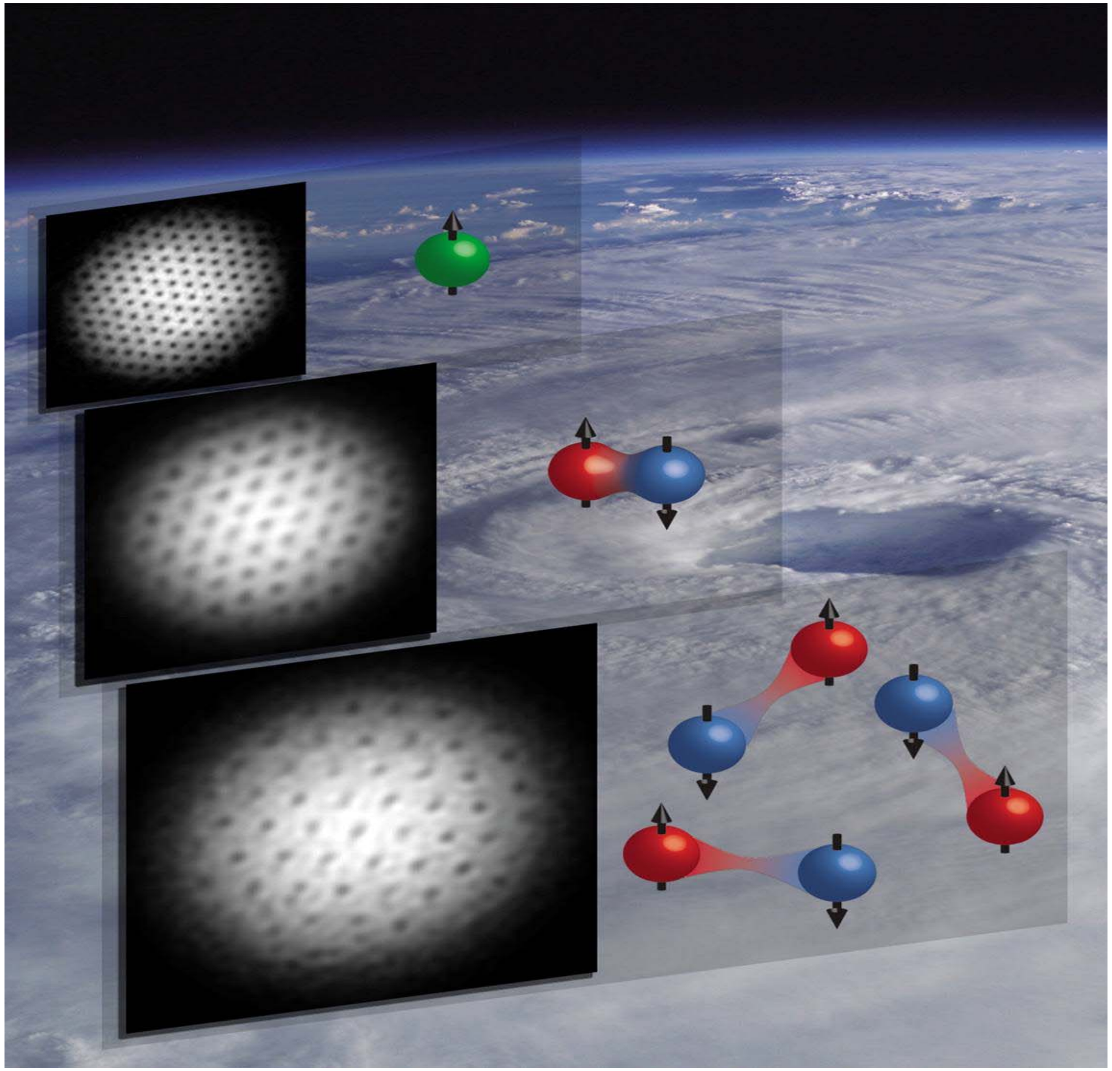


Gallery of superfluid gases (Ketterle's)

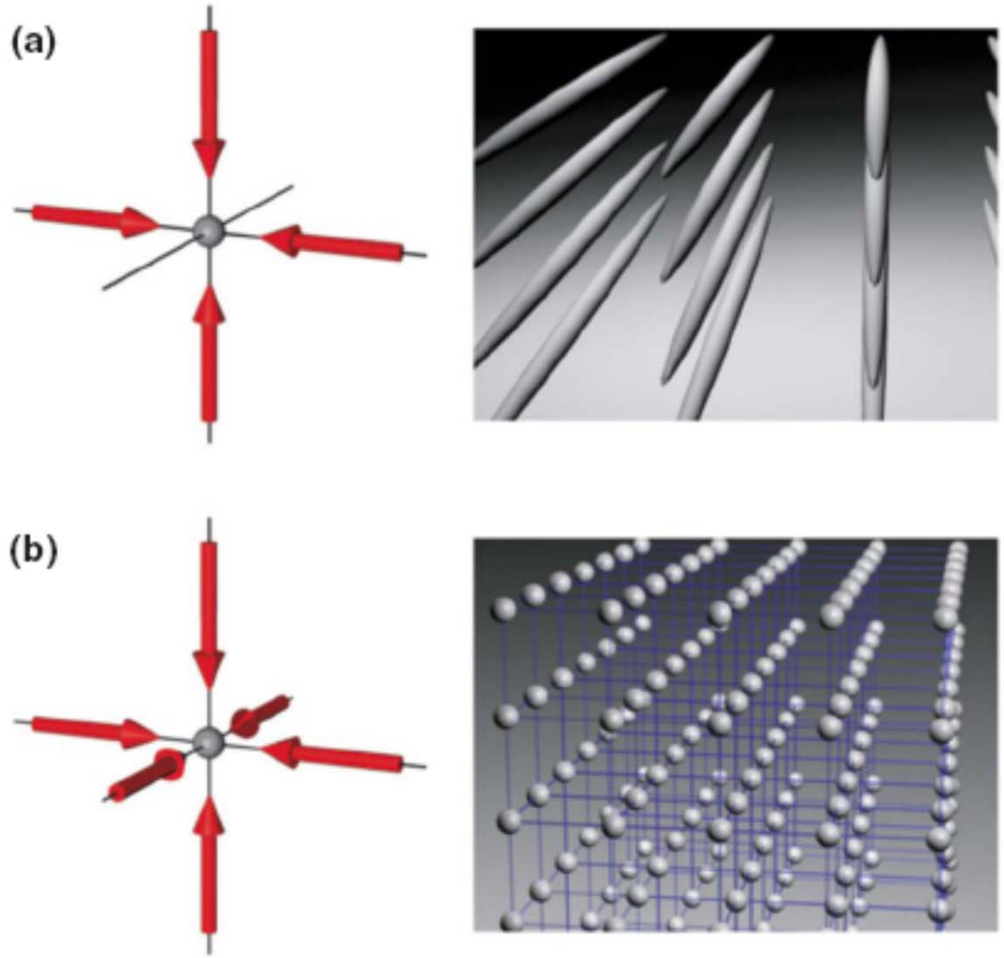
Atomic Bose-Einstein condensate (sodium)

Molecular Bose-Einstein condensate (lithium $^6Li_2$)

Pairs of fermionic atoms (lithium ^6Li)



Optical Lattices



Crystal = Lattices + Atoms

Optical lattice

The simplest possible periodic optical potential is formed by overlapping two counter-propagating beams. This results in a standing wave

$$E(z) = E_0 \sin(kz + \theta) \cos \omega t$$

Averaging over fast optical oscillations (AC Stark effect) gives

$$V(z) = -V_0 \sin^2(kz + \theta)$$

Combining three perpendicular sets of standing waves we get a simple cubic lattice

$$V(r) = -V_0 \cos qx - V_0 \cos qy - V_0 \cos qz$$

This potential allows separation of variables

$$\Psi(x, y, z) = \psi(x)\, \psi(y)\, \psi(z)$$

Ultracold atoms
A “toolbox” for designing Hamiltonian

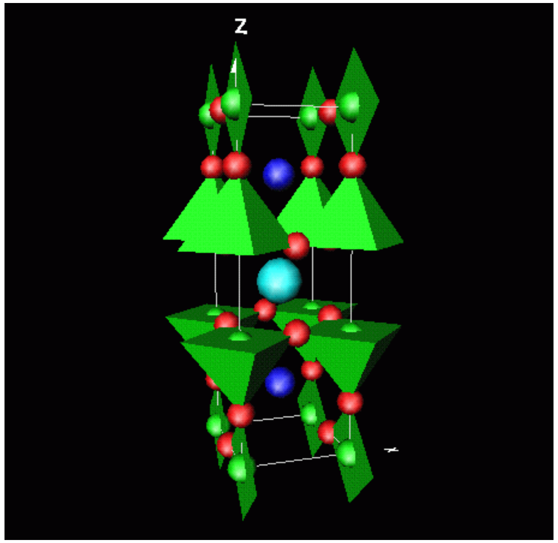
Systems of cold atoms and molecules can be used for engineering and manipulation of strongly correlated quantum states. New phenomena (Hamiltonians) may be realized.

- **Simulating fundamental models such as Hubbard model**
- **Understanding quantum magnetism (existence of spin liquid) and unconventional fermion pairing**

Interdisciplinary Study between cold atoms & condensed matter physics

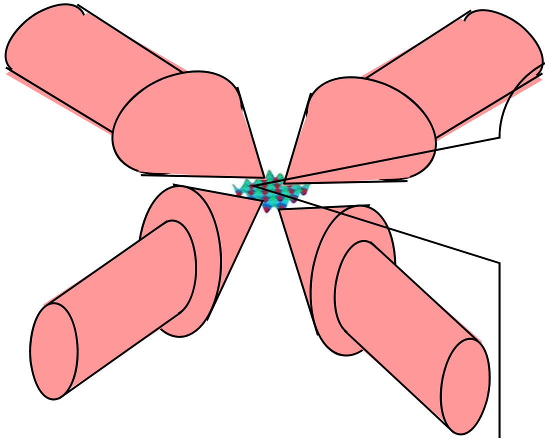
Quantum Simulations

Bosons, Fermions, Boson-Fermion Mixtures
Hamiltonian Engineering

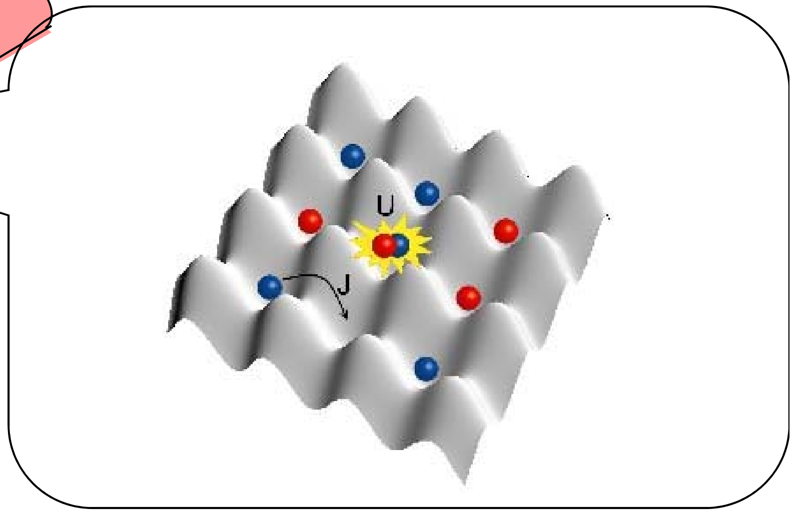


$YBa_2Cu_3O_7$

Antiferromagnetic and superconducting Tc of the order of 100 K



same microscopic model



Atoms in optical lattice

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

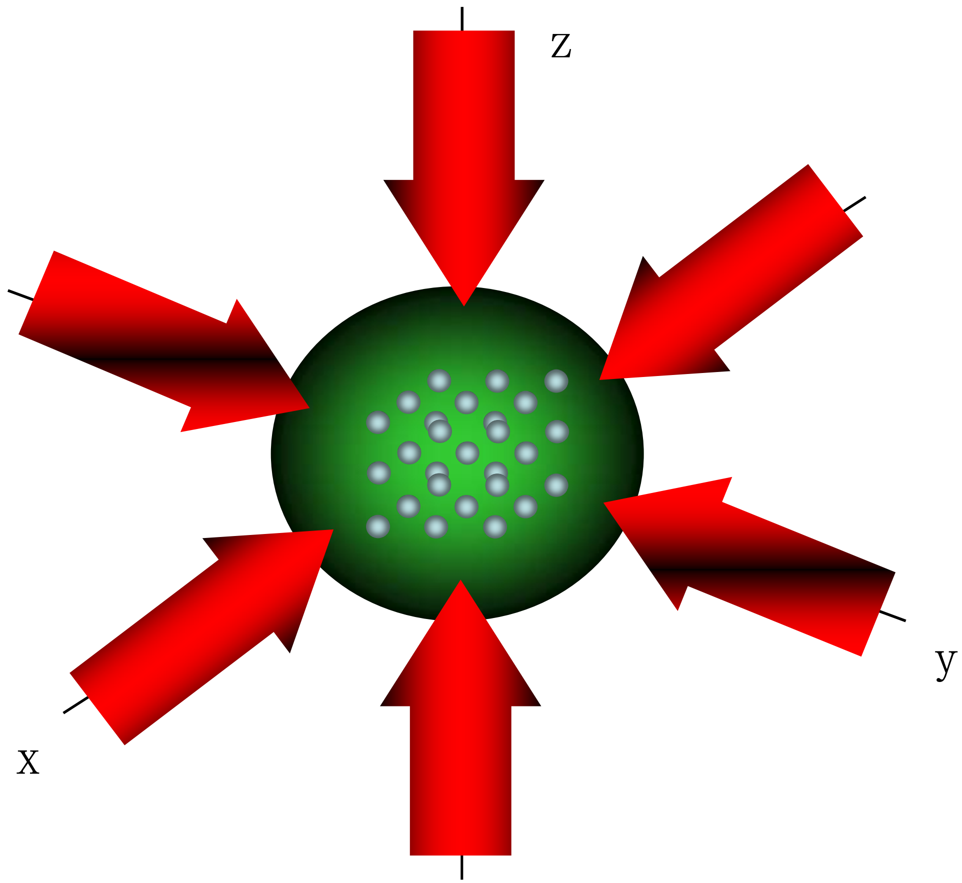
$$\mathcal{H} = -t \sum_{\langle ij \rangle\, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$

Well controlled system

Quantity	Superconductor	Fermi gases
Vortices	B field	Rotation
Fermi energies	B field	Rf control of populations
Interaction strength	Change sample	B field near Feshbach resonances

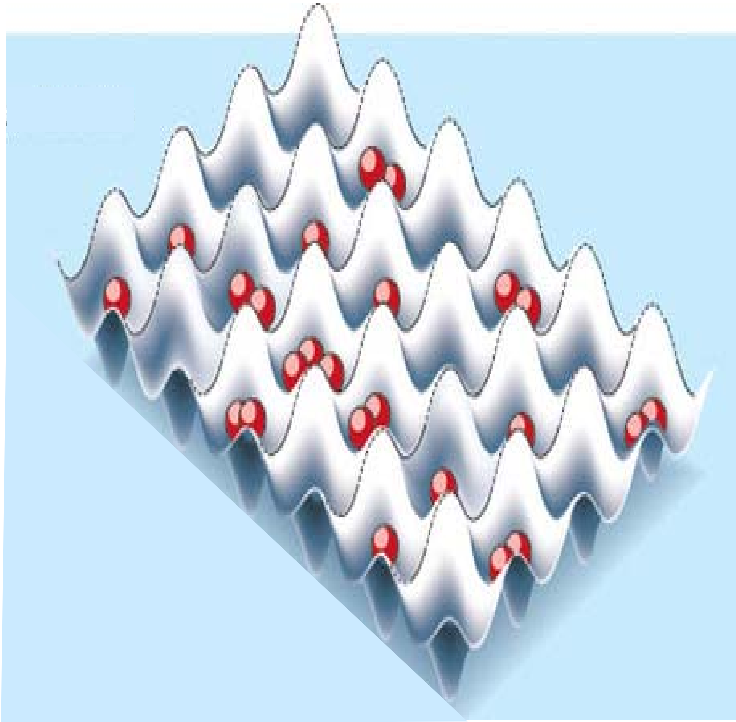
2. Ultracold atoms in Optical Lattices

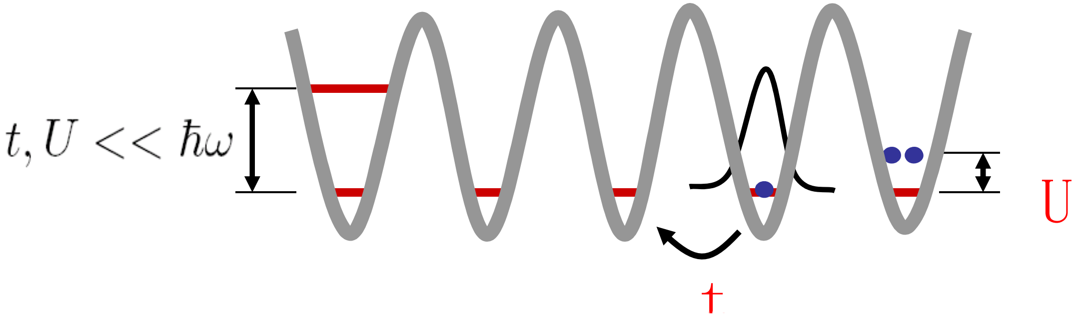
1) Bosons in Optical Lattices
2) Fermions in Optical Lattices
3) Hamiltonian Engineering



2.1 Bosons in Optical Lattices

Bose-Hubbard model





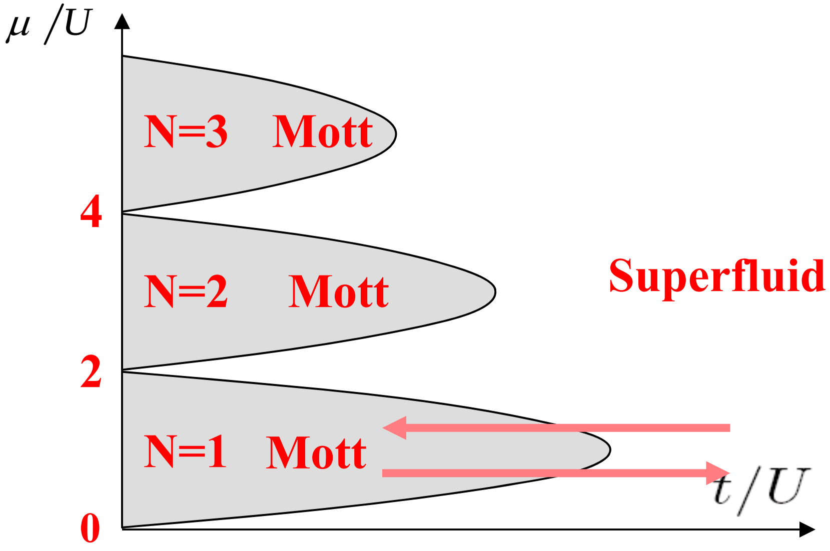
$$\mathcal{H} = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

t – tunneling of atoms between neighboring wells

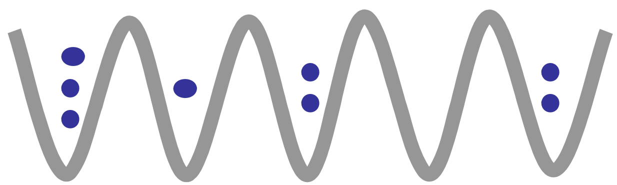
U – repulsion of atoms sitting in the same well

Apply a periodic potential (standing laser beams) to trapped ultracold bosons (^{87}Rb)

Mean-field phase diagram



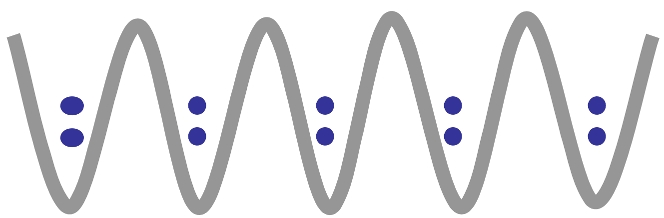
M.P.A. Fisher et al., PRB 40, 546 (1989)



$$U << Nt$$

Superfluid phase

Weak interactions

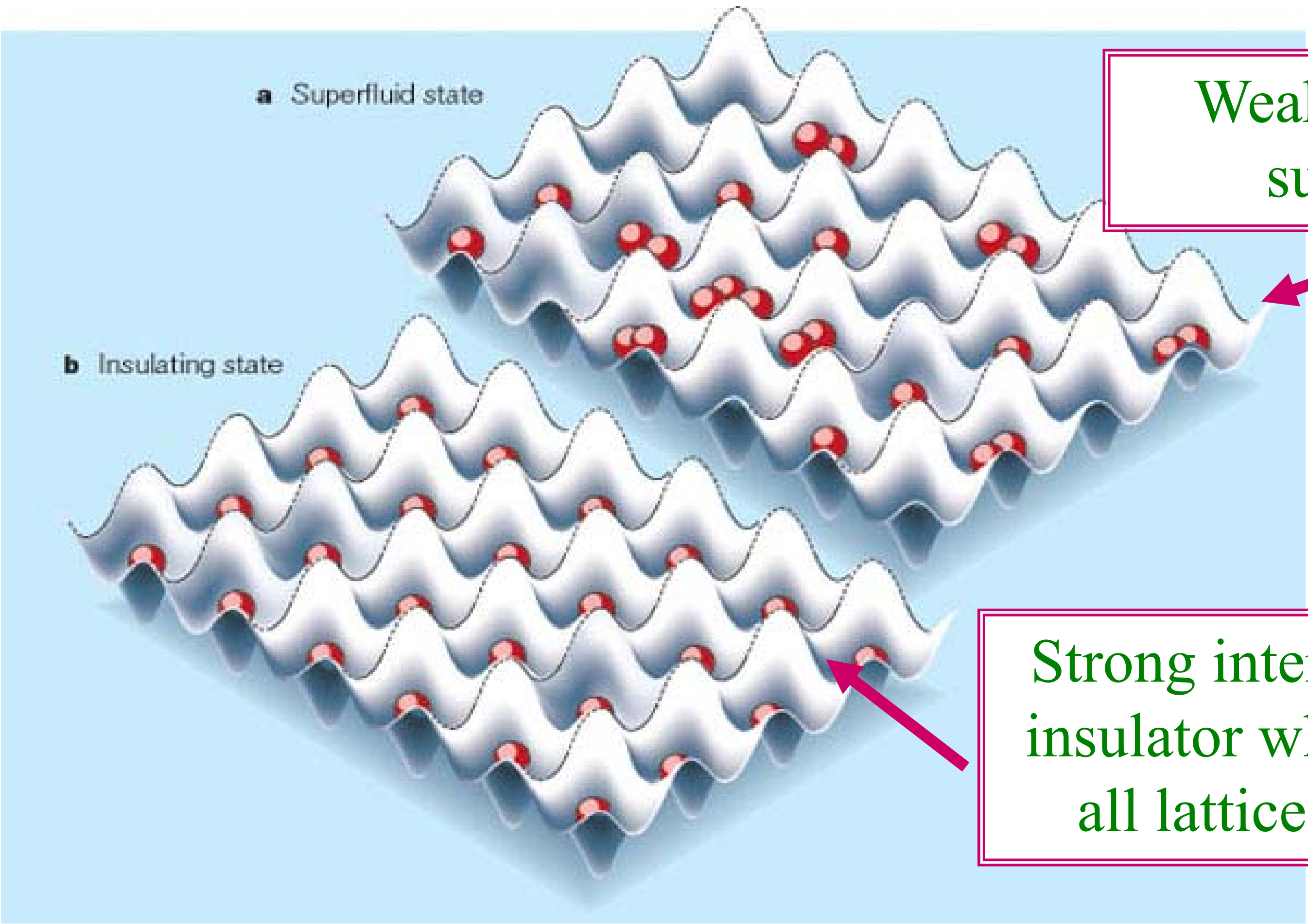


$$U >> Nt$$

Mott insulator phase

Strong interactions

Schematic Configurations

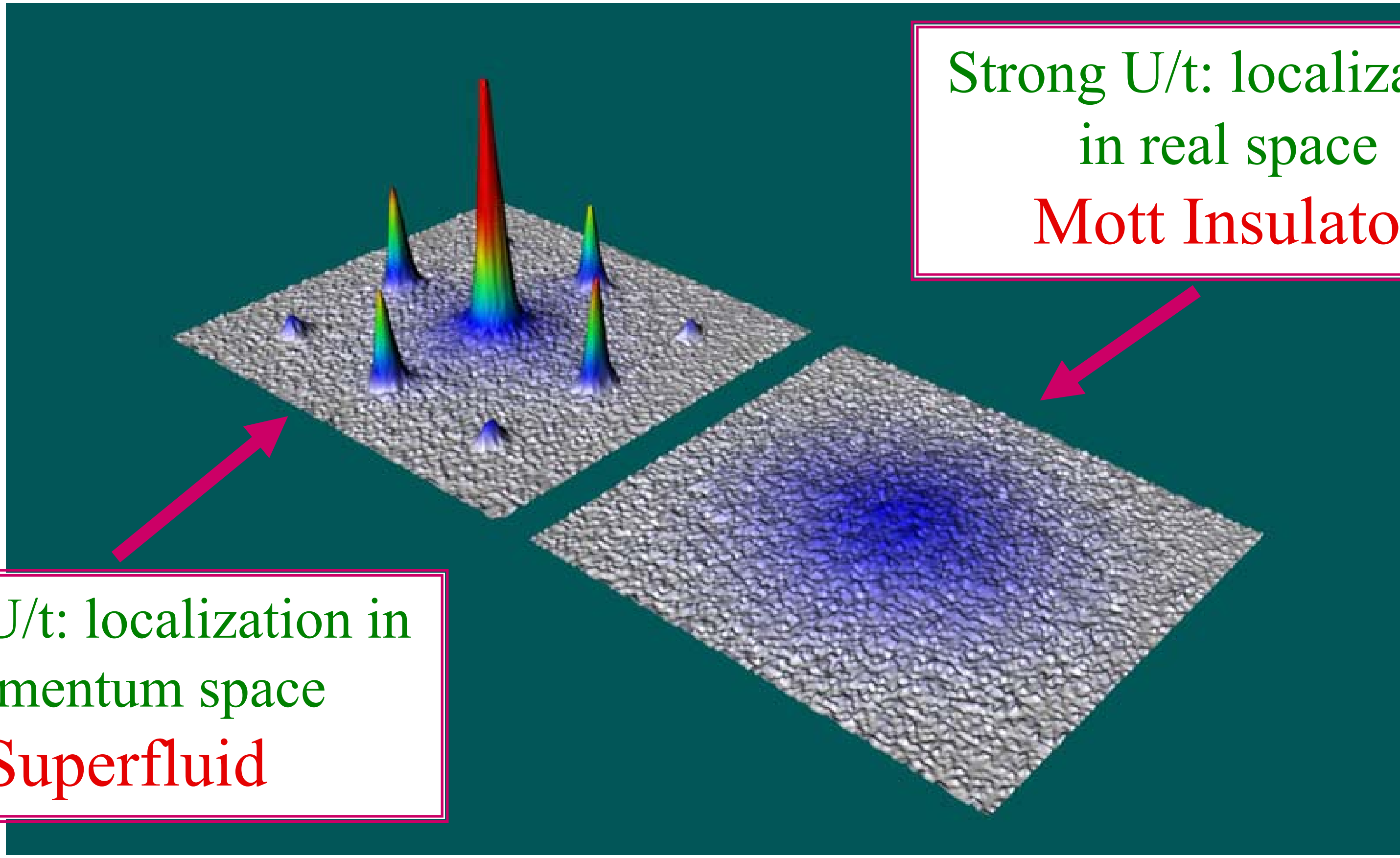


Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* 415, 39 (2002).

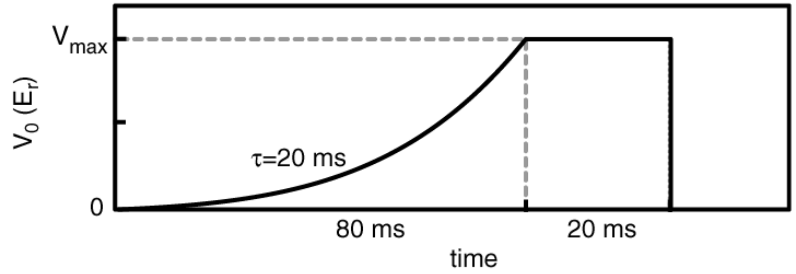
Experimental Verifications



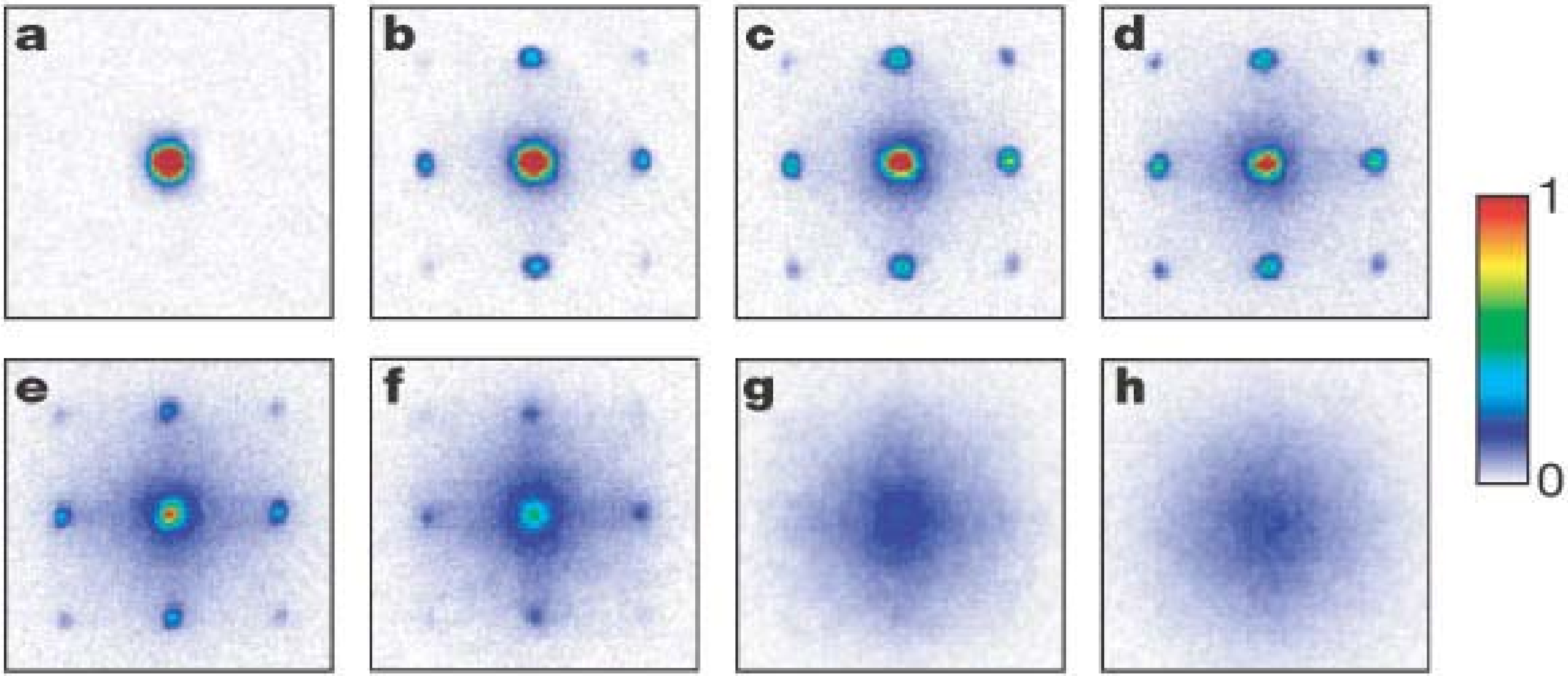
Momentum distribution function of bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* 415, 39 (2002).

S-I QPT



$V_0=0E_r$ $V_0=3E_r$ $V_0=7E_r$ $V_0=10E_r$



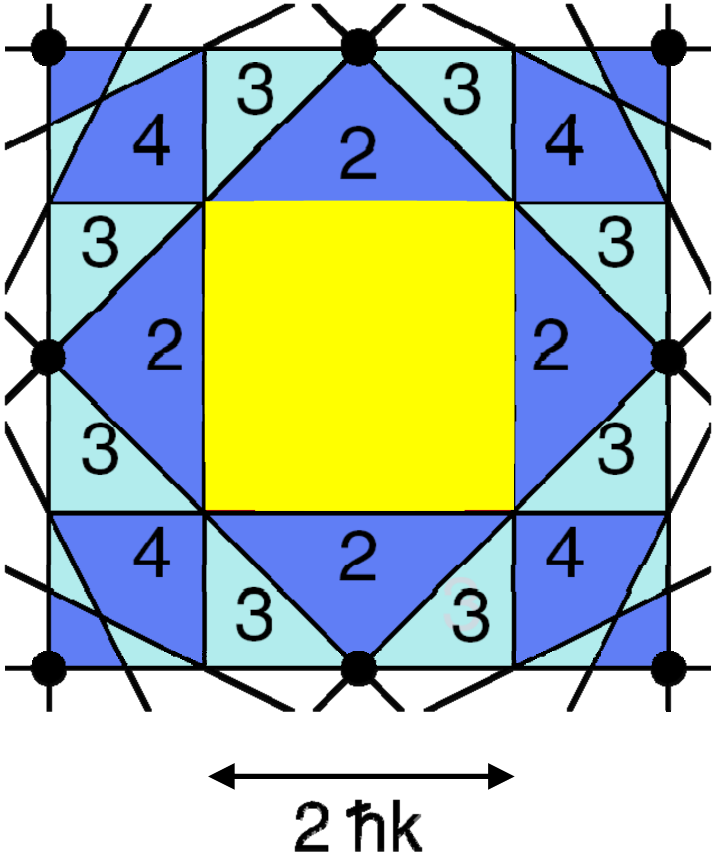
$V_0=13E_r$ $V_0=14E_r$ $V_0=16E_r$ $V_0=20E_r$

2.2 Fermions in Optical Lattices

Filling the Brillouin zone

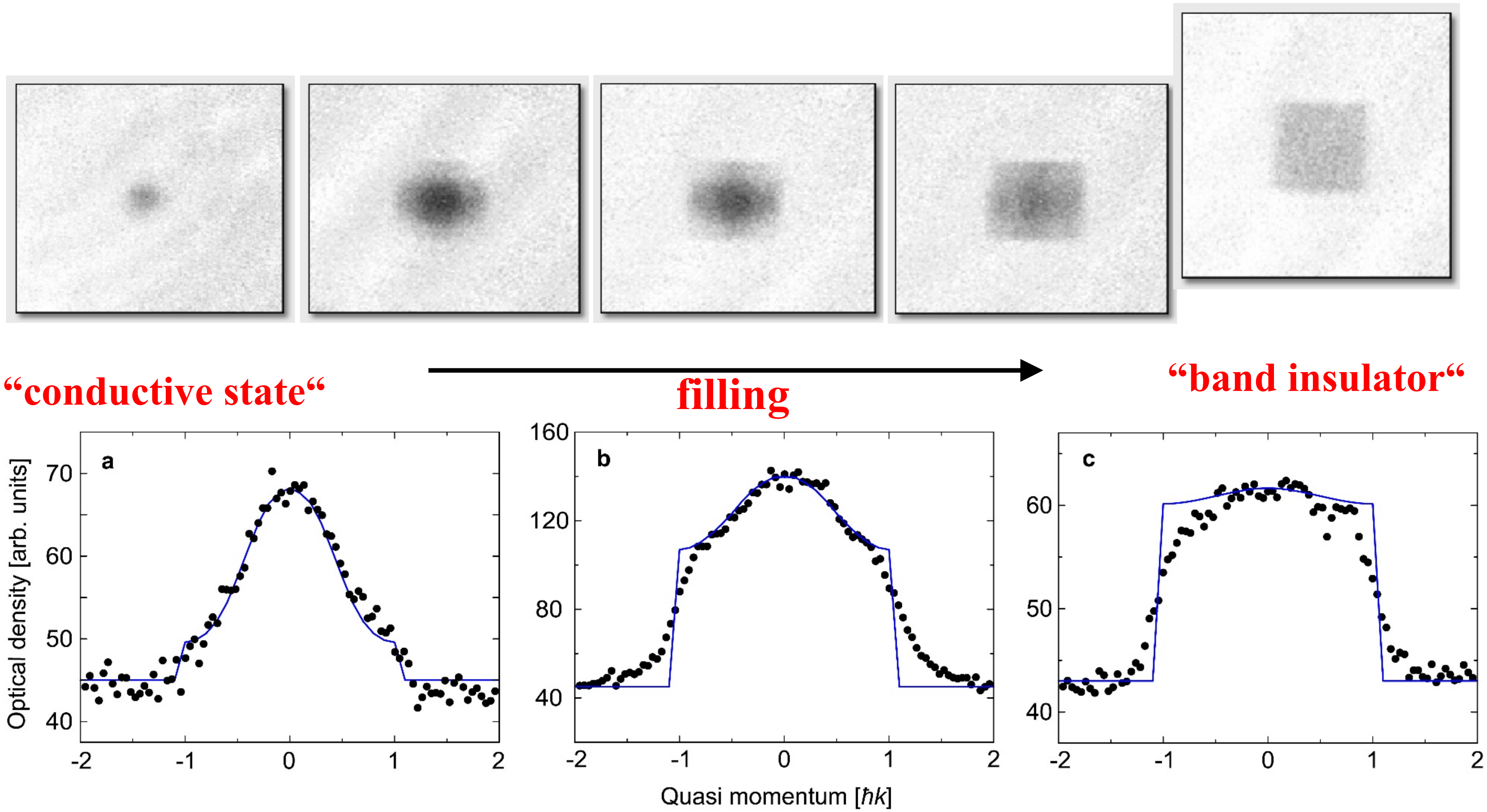
Brillouin zones
of a square lattice

Fermi
surface



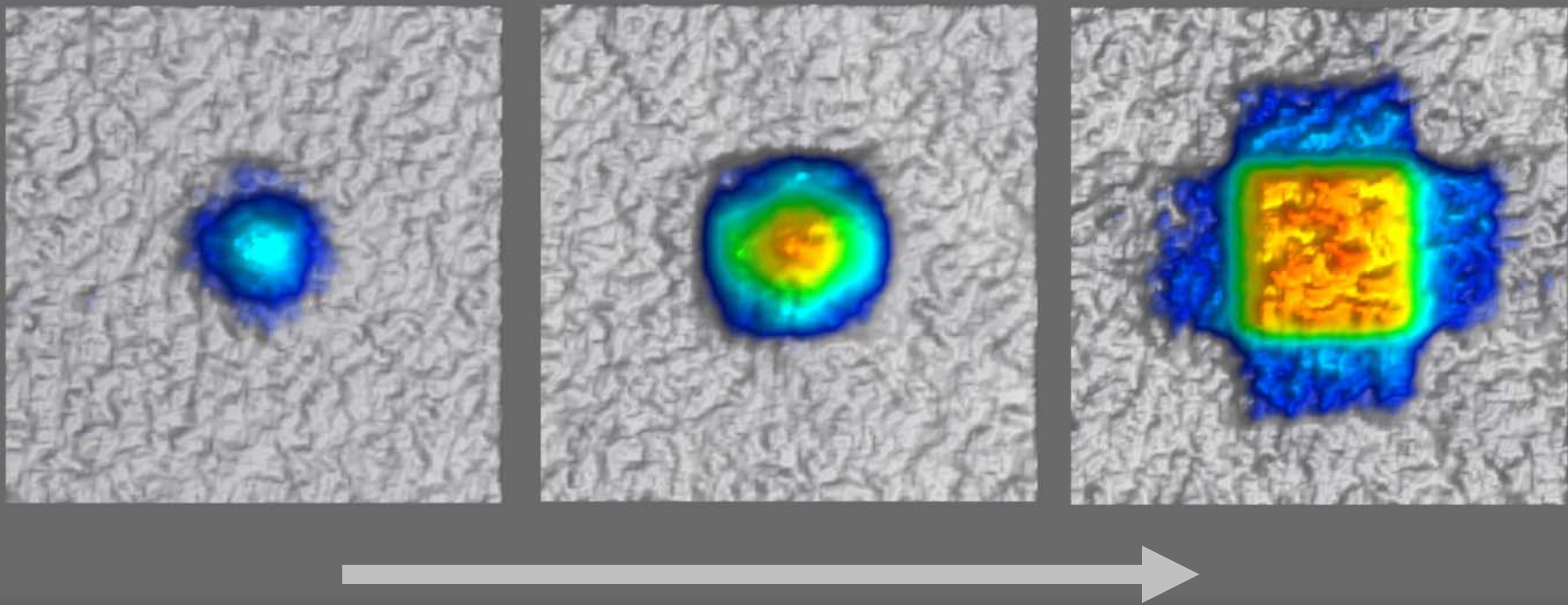
quasi momentum

Experimental Fermi surfaces



M. Köhl, H. Moritz, T. Stöferle, K. Günter and T. Esslinger, Phys. Rev. Lett. 94, 080403 (2005).

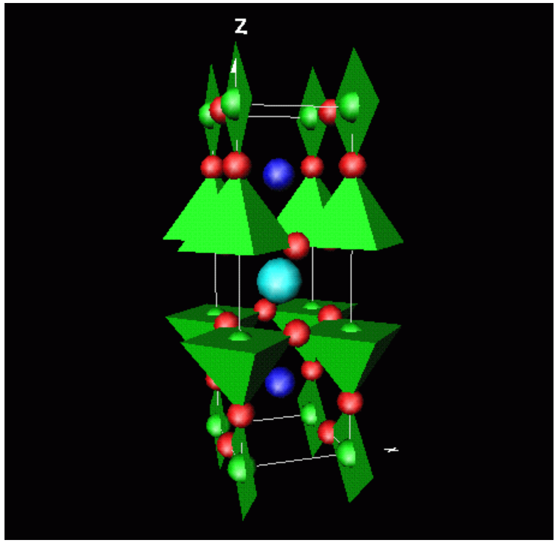
Fermi Surfaces

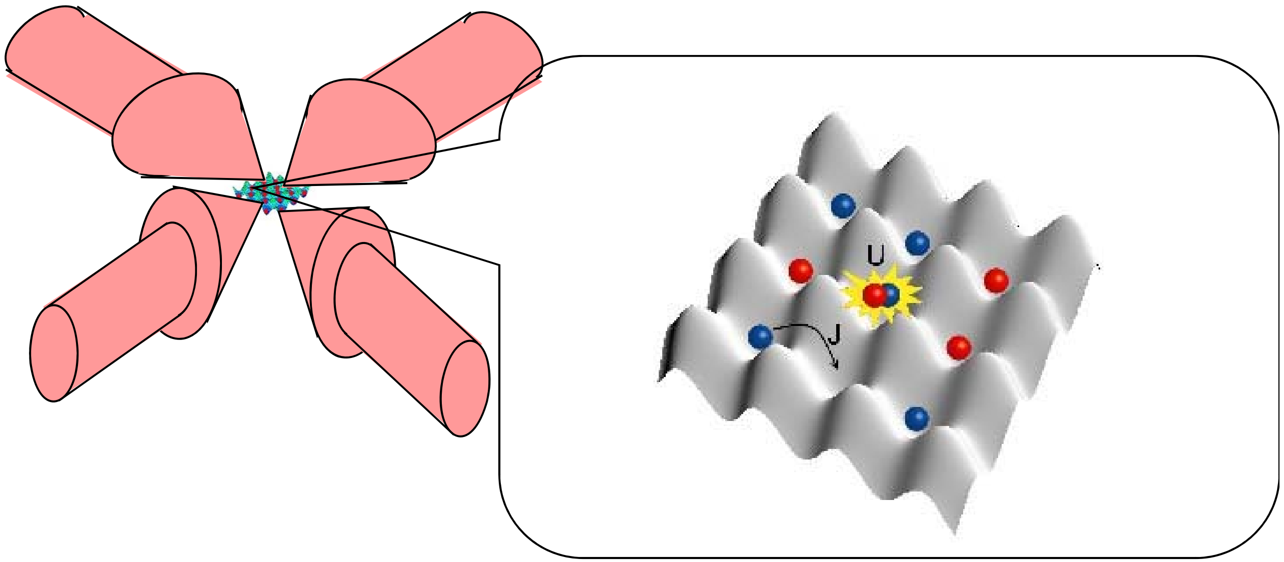


Increasing number of atoms in the lattice

2) Fermionic Hubbard model

Quantum simulations with ultracold atoms





$YBa_2Cu_3O_7$

Atoms in optical lattice

Antiferromagnetic and superconducting Tc of the order of 100 K

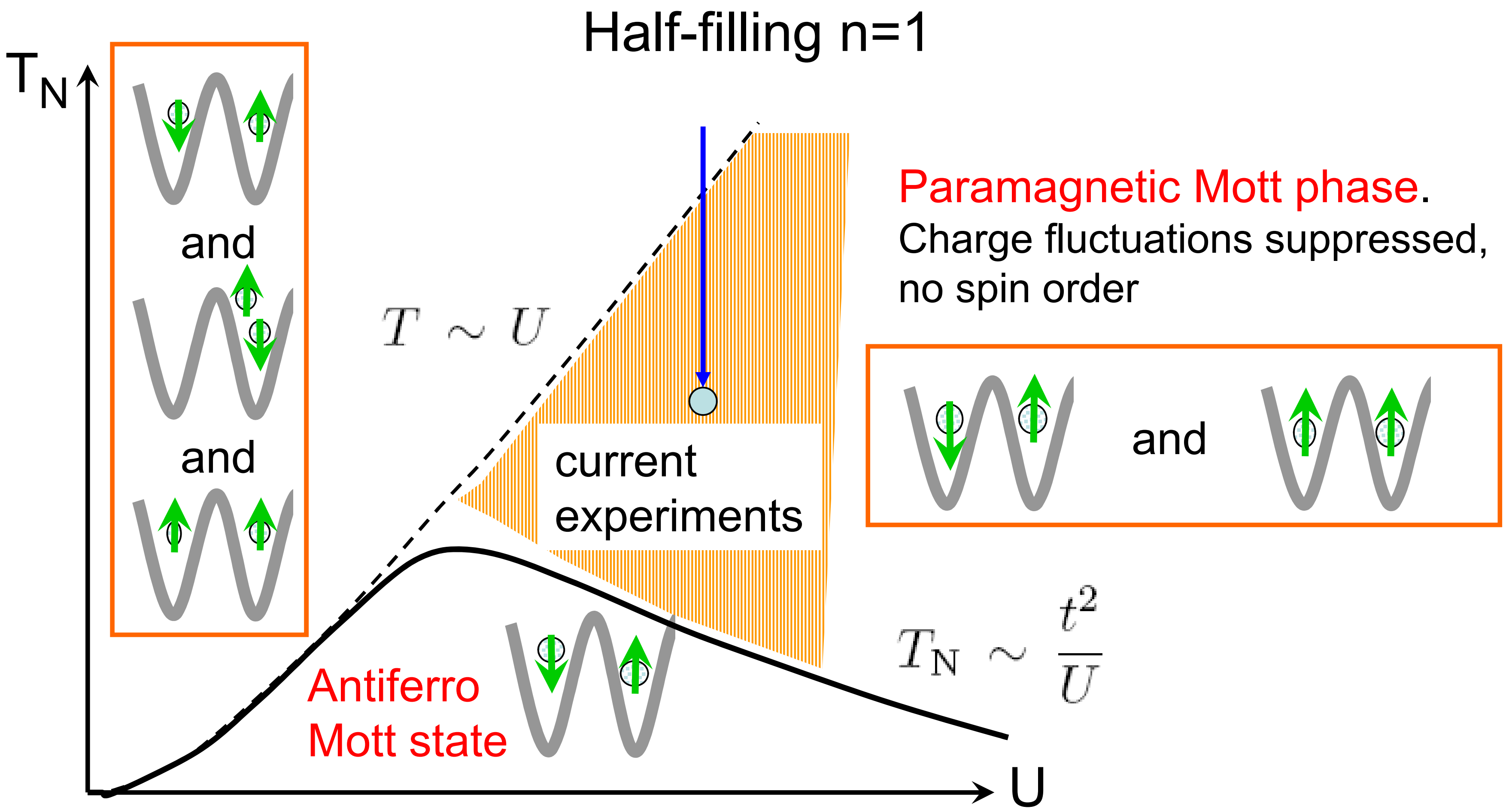
same microscopic model

Antiferromagnetism and pairing at sub-micro Kelvin temperatures

$$\mathcal{H} = -t \sum_{\langle ij \rangle \, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i$$

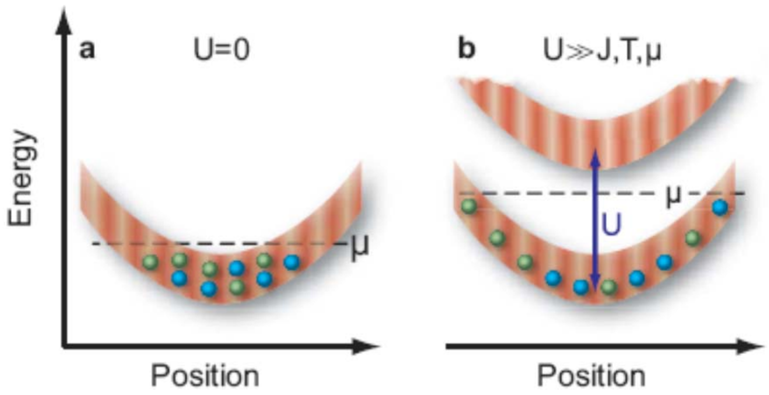
Energy scales of the half-filled Hubbard model

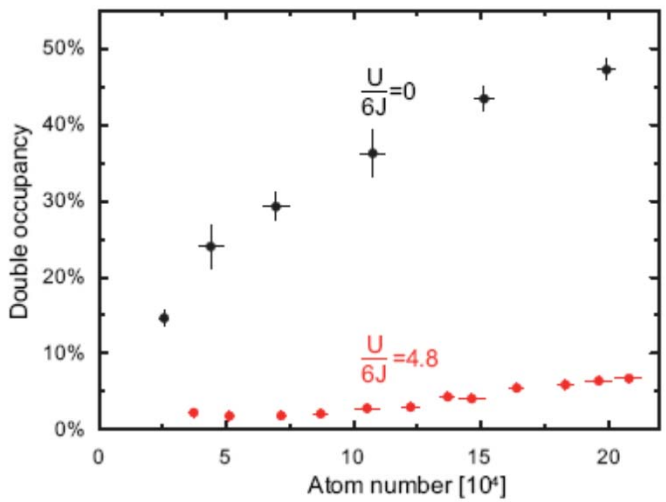
$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$



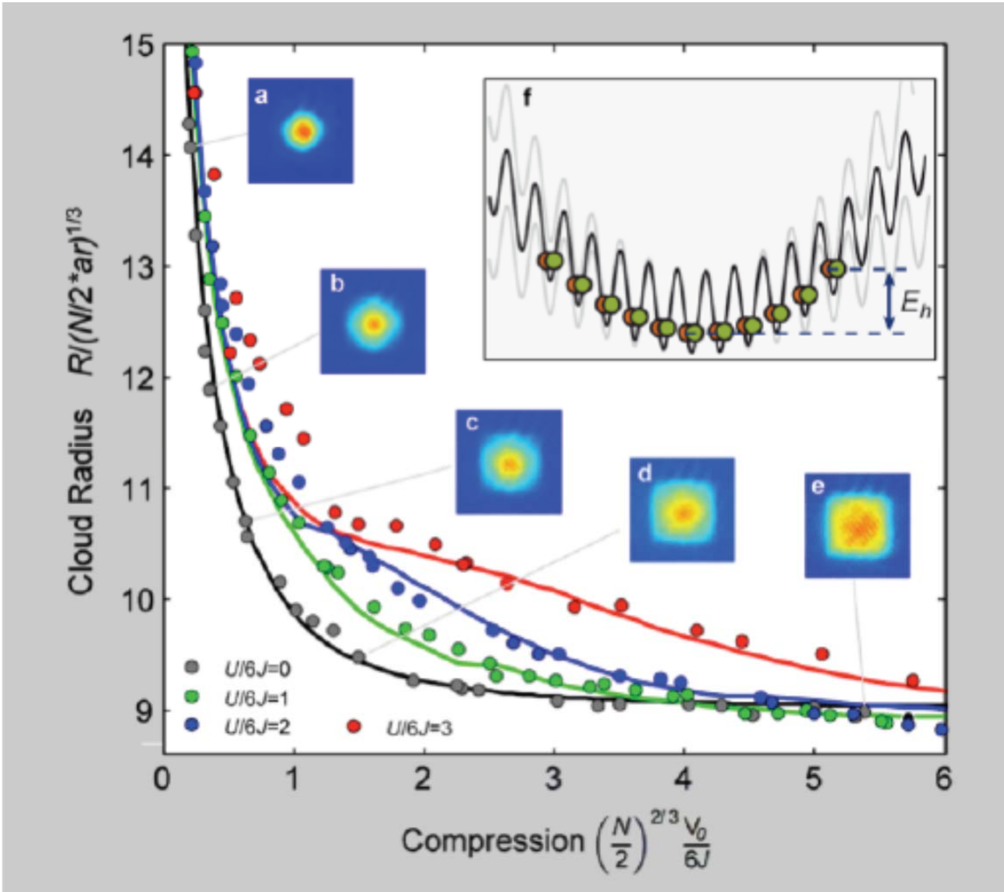
Signatures of incompressible Mott state of fermions in optical lattice

Suppression of double occupancies
R. Joerdens et al., Nature (2008)





Compressibility measurements
U. Schneider et al., Science (2008)



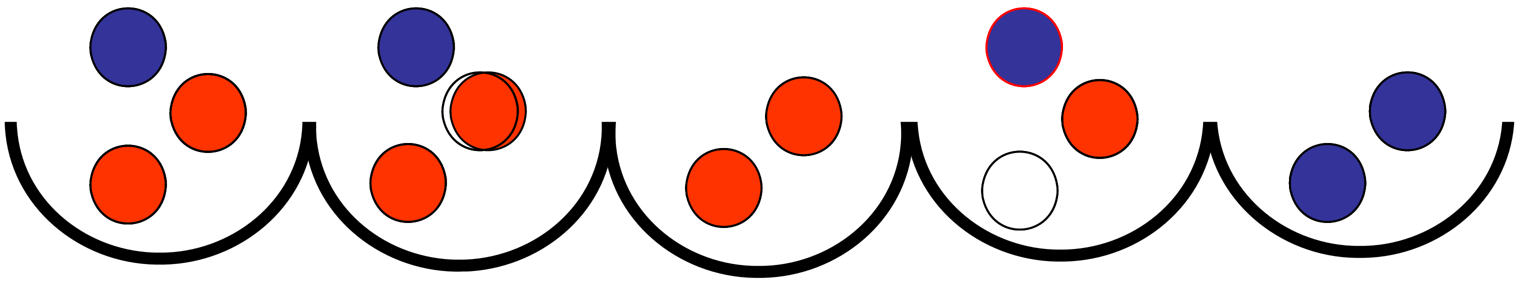
2.3 Bose-Fermi Mixtures in Optical Lattices

- **Fermions and bosons on equal footing in a lattice:**
- **Atomic physics “beats” condensed matter physics!!!**

- **Novel quantum phases and novel kinds of pairing:**
  Fermion-boson pairing!!!

- **Novel possibilites of control of the system**

Quantum phases of the Bose-Fermi Hubbard model



Description: **Bose-Fermi Hubbard model**

i) Phase I – Mott (n) plus Fermi gas of fermions with NN interactions
ii) Phase II – Interacting composite fermions (fermion + bosonic hole)
iii) Phase III – Interacting composite fermions (fermion + 2 bosonic holes)

$$H = -J^b \sum_{<ij>} b_i^+ b_j + J^f \sum_{<ij>} f_i^+ f_j + h.c. + \frac{1}{2}\sum_i U n_i (n_i - 1) + \sum_i V n_i m_i - \sum_i \mu_i n_i$$

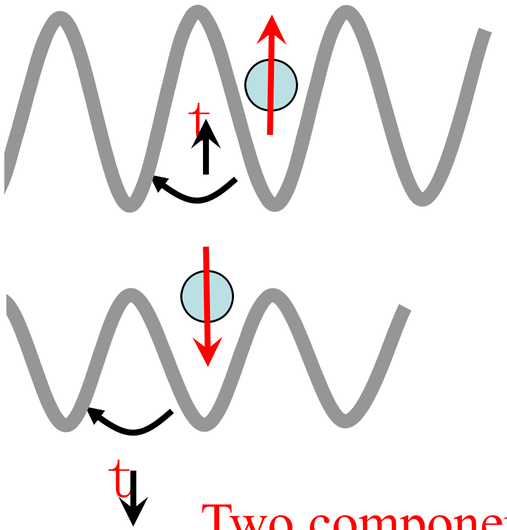
Lewenstein et al. PRL (2003), Ferhman et al. Optics Express (2004)

2.4 Designing & Engineering Hamiltonian

Hamiltonian Engineering (I)

Two component Bose mixture (different hyperfine states) in optical lattice

$^{87}\mathrm{Rb}$



Example: Mandel et al., Nature (2003)

$$| \uparrow \rangle = | F = 1, m_F = -1 \rangle$$

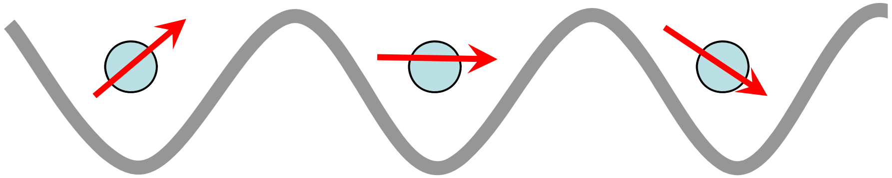
$$| \downarrow \rangle = | F = 2, m_F = -2 \rangle$$

Two component Bose Hubbard model

$$\mathcal{H} = - t_\uparrow \sum_{\langle ij \rangle} b^\dagger_{i\uparrow} b_{j\uparrow} - t_\downarrow \sum_{\langle ij \rangle} b^\dagger_{i\downarrow} b_{j\downarrow} + U_{\uparrow\uparrow} \sum_i n_{i\uparrow}(n_\uparrow - 1)$$

$$+ U_{\downarrow\downarrow} \sum_i n_{i\downarrow}(n_\downarrow - 1) + U_{\uparrow\downarrow} \sum_i n_{i\uparrow} n_\downarrow$$

Hamiltonian Engineering (II)



Kuklov and Svistunov, PRL (2003)

Duan et al., PRL (2003)

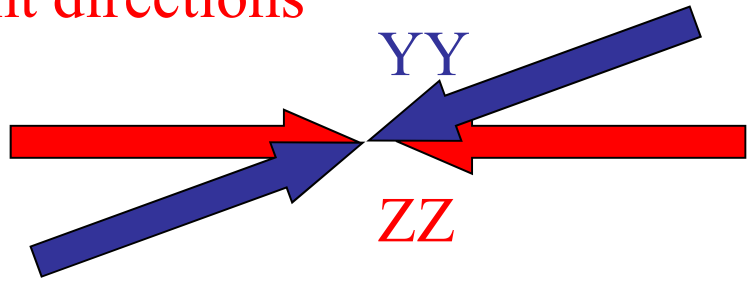
$$\mathcal{H} = J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + J_\perp \sum_{\langle ij \rangle} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right)$$

$$J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_{\uparrow\uparrow}} - \frac{t_\downarrow^2}{U_{\downarrow\downarrow}} \qquad J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}$$

- Ferromagnetic $U_{\uparrow\downarrow} >> U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$
- Antiferromagnetic $U_{\uparrow\downarrow} << U_{\uparrow\uparrow},\ U_{\downarrow\downarrow}$

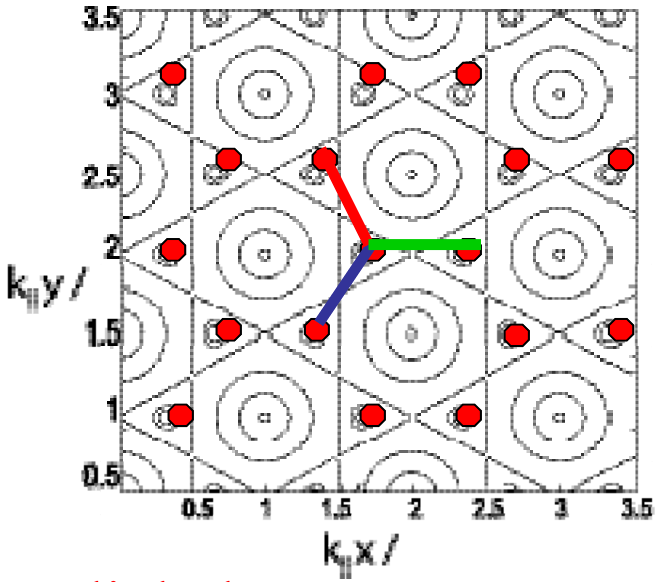
Hamiltonian Engineering (III)

Optical lattice in 2 or 3 dimensions:
polarizations & frequencies of standing waves can be different for different directions



• Example: exactly solvable model, Kitaev (2002), honeycomb lattice with

$$H = J_x \sum_{<i,j>\in x} \sigma_i^x \sigma_j^x + J_y \sum_{<i,j>\in y} \sigma_i^y \sigma_j^y + J_z \sum_{<i,j>\in z} \sigma_i^z \sigma_j^z$$

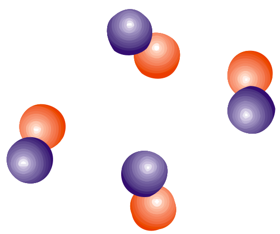


Can be created with 3 sets of standing wave light beams !

Non-trivial topological order is expected to be detected in such a controlled experiments

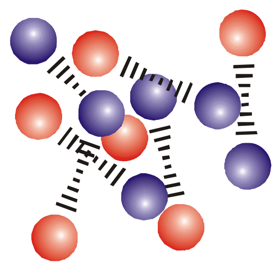
3. BEC-BCS Crossover in Ultracold Fermi Gases

Molecules of
fermionic atoms



BEC of weakly
bound molecules

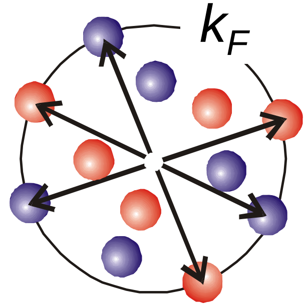
Generalized Cooper
pairs of fermionic
atoms



**BCS - BEC
crossover**

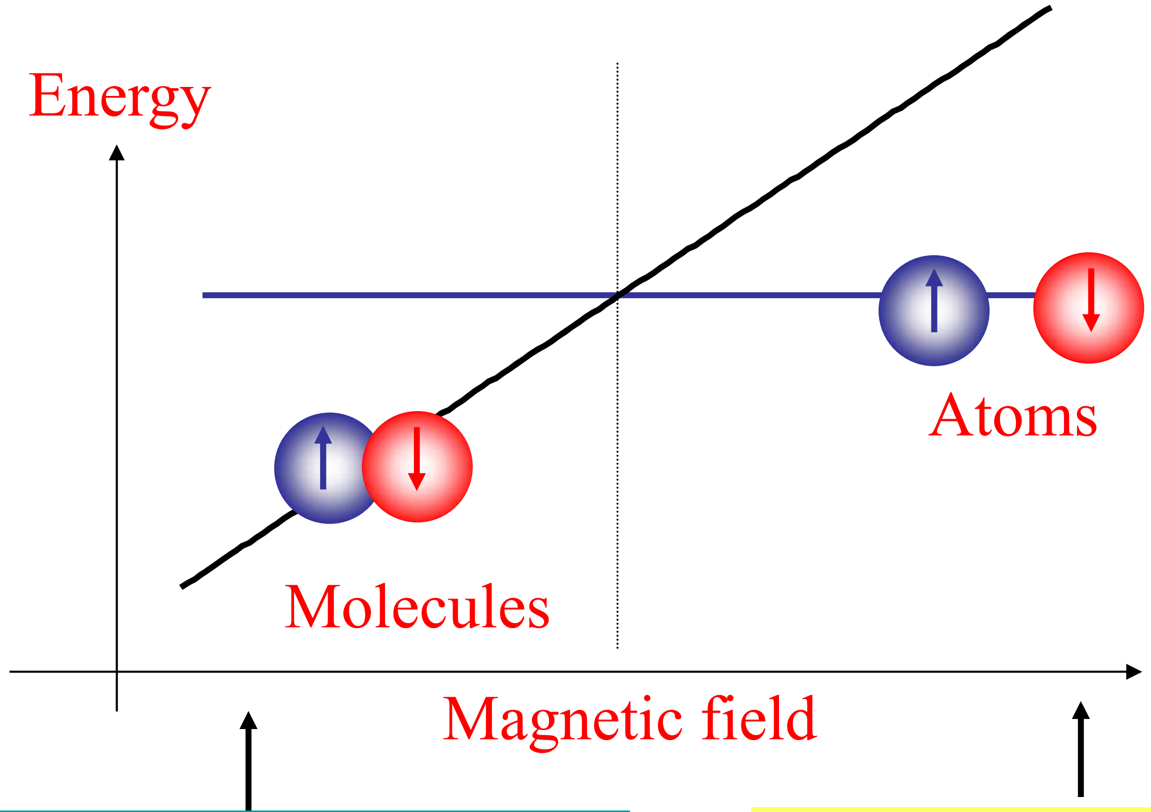
Cooper pairs

spin ↑
spin ↓



BCS superconductivity
Cooper pairs: correlated
momentum-space pairing

Many thoreies on BCS-BEC crossover for example:
Eagles, Leggett, Nozieres and Schmitt-Rink, Randeria, Strinati, Zwerger, Holland, Timmermans, Griffin, Levin …



Atoms form stable molecules

Atoms repel each other

a>0

BEC of Molecules:
Condensation of
tightly bound fermion pairs

Molecules are unstable

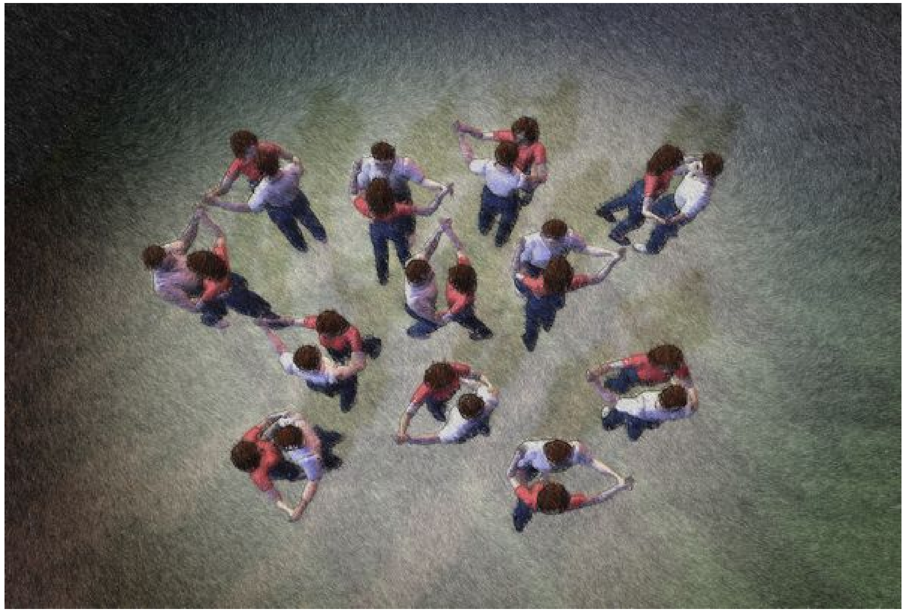
Atoms attract each other

a<0

BCS-limit:
Condensation of
long-range Cooper pairs

BCS-BEC Crossover

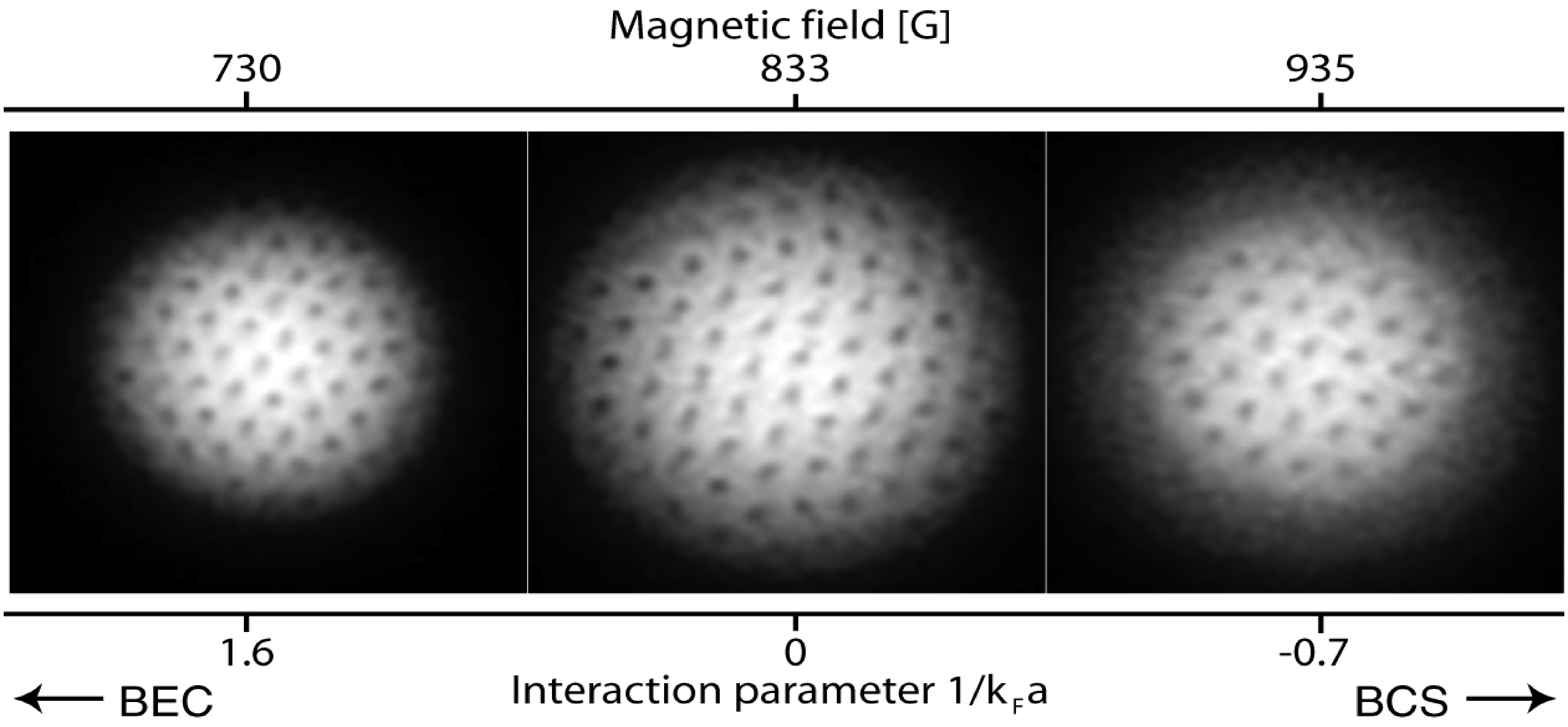
BCS– weak attraction



BEC– strong attraction

Tunable attractive interaction

Optimized vortex lattices in the crossover



M.W. Zwierlein, …, W. Ketterle, Nature 435, 1047-1051 (2005)

The End

Thanks for your attention!!!