

Recent development of quantum spin liquids

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Opportunity for students and postdocs

- My research group is looking for graduate students and postdocs
- Our **postdocs and visiting professors** are generously funded.



Some kind of center is forming. we are hiring people for all field of physics, and theoretical physics.

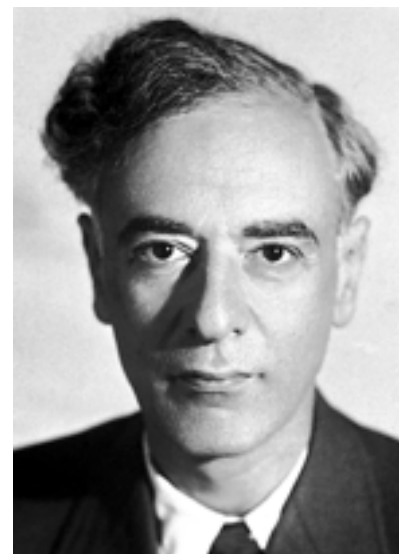
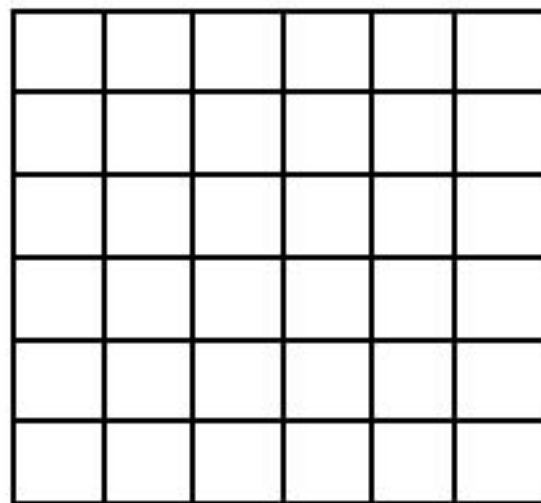
Outline

- A general introduction to quantum spin liquids
- Spinon Fermi surface $U(1)$ quantum spin liquid
- Rare earth triangular lattice quantum spin liquid and experiment prediction
- Control spinons in a quantum spin ice $U(1)$ quantum spin liquid

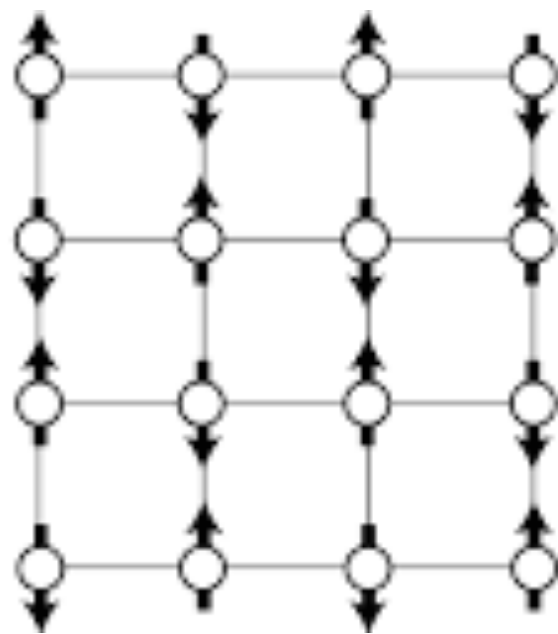
Neel vs Landau (1930-40s)



Neel



Landau



$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$\text{Spin singlet} = \frac{1}{\sqrt{2}} \left[\begin{array}{c} \uparrow \\ \text{red} \end{array} \text{---} \begin{array}{c} \downarrow \\ \text{blue} \end{array} - \begin{array}{c} \downarrow \\ \text{red} \end{array} \text{---} \begin{array}{c} \uparrow \\ \text{blue} \end{array} \right]$$



P. W. Anderson

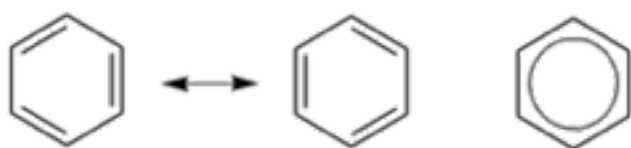
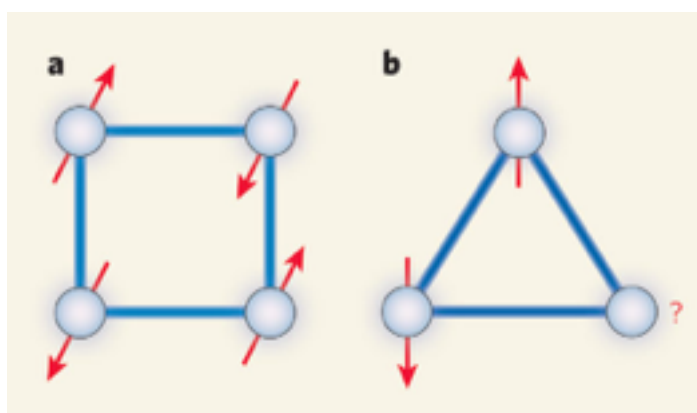
The idea of quantum spin liquid (1973)

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

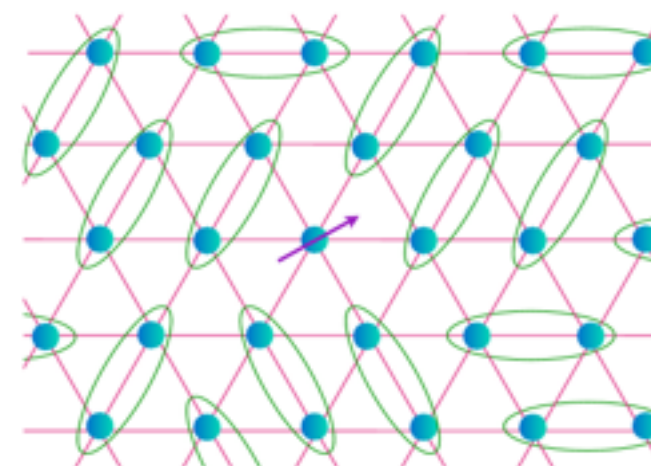
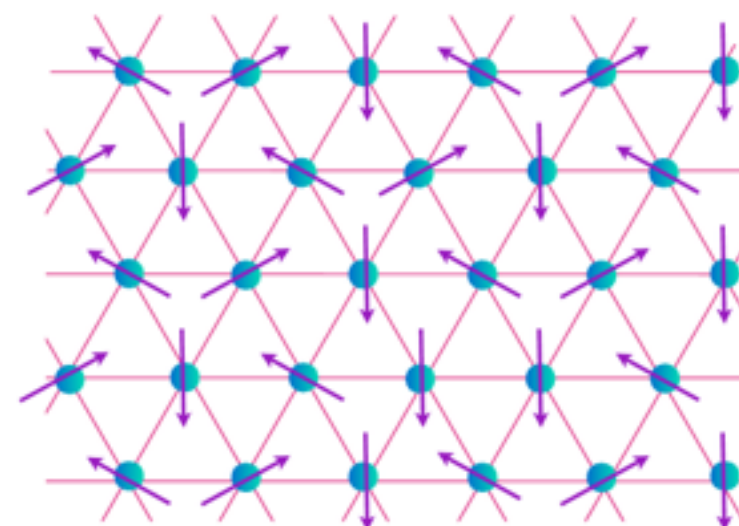
P. W. Anderson

type would be insulating; it would represent an alternative state to the Néel antiferromagnetic state for $S = 1/2$. An estimate of

it is very il
motivation

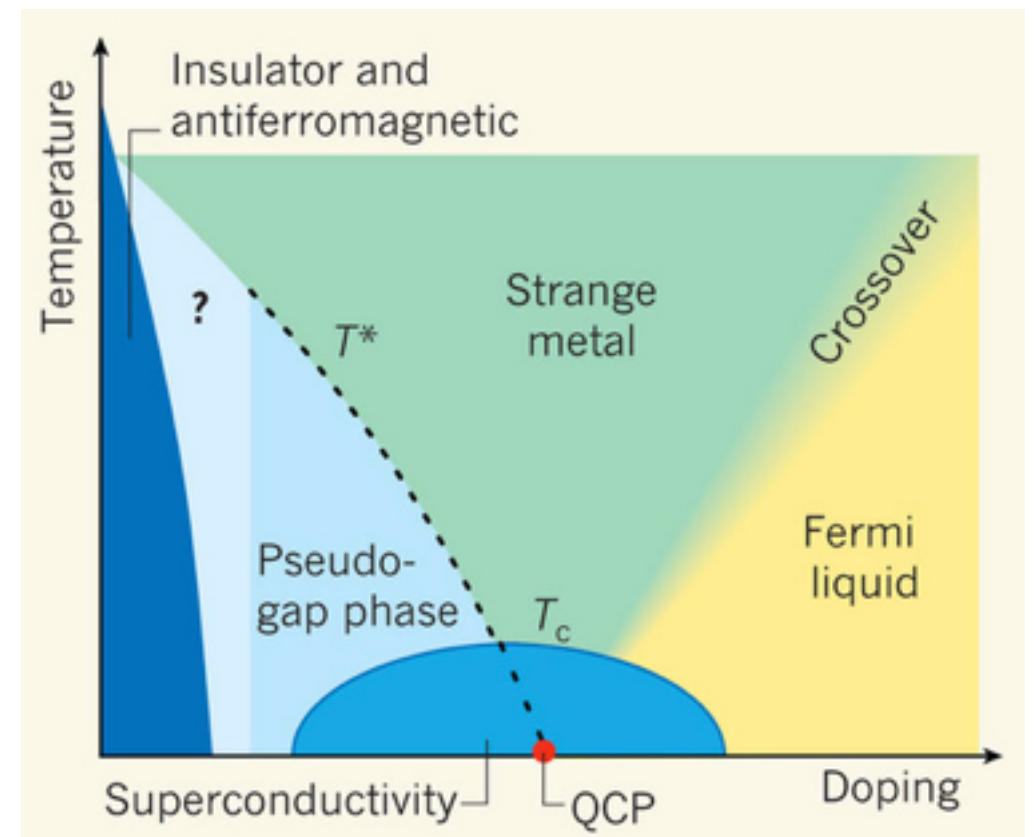
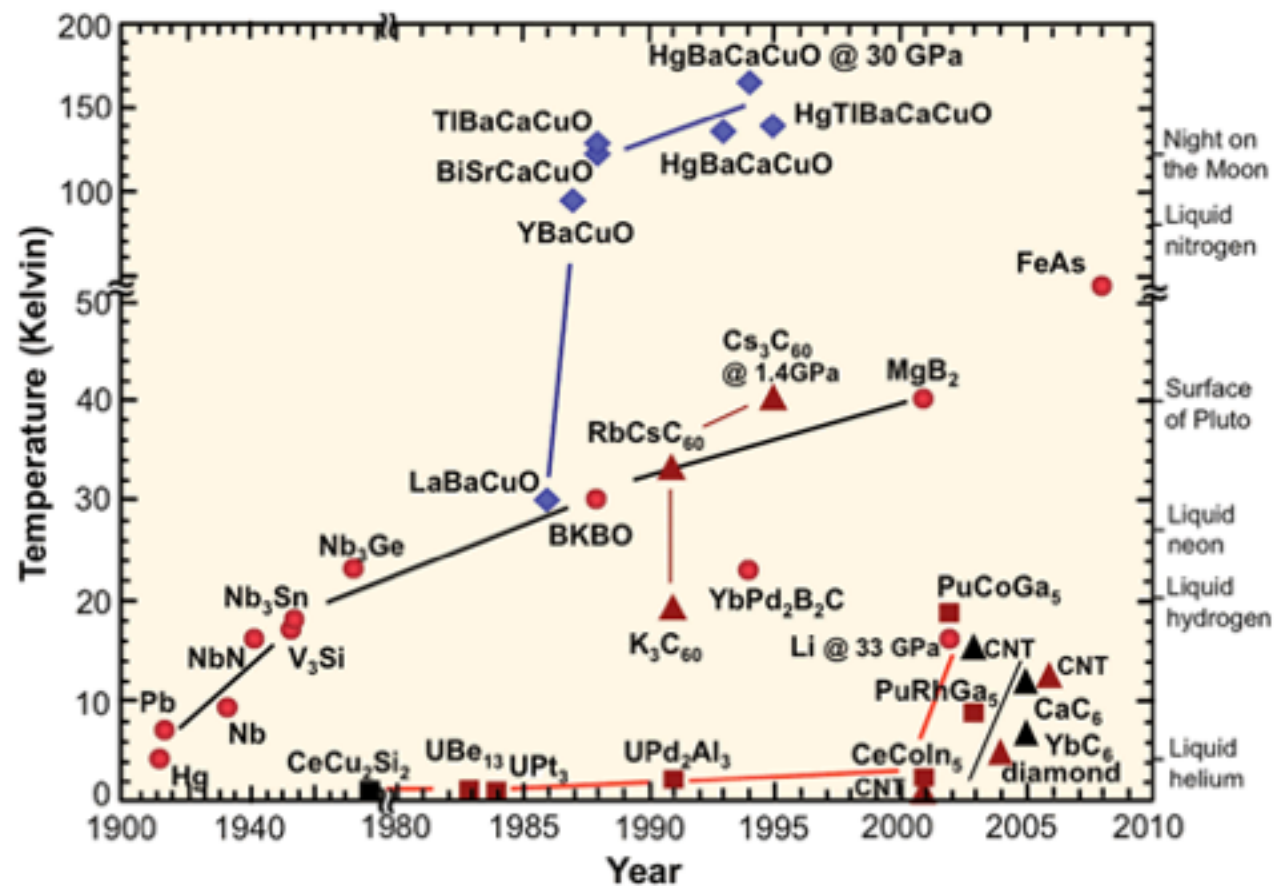


Pauling's RVB wavefunction
for Benzene molecule

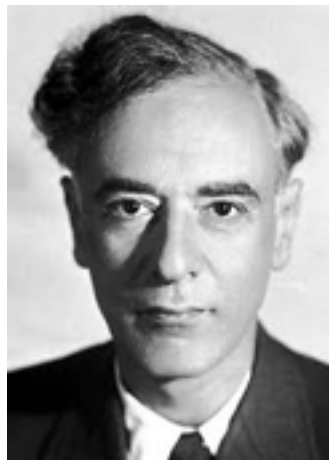


$$\text{Diagram of a pair of sites in a green oval} = \frac{1}{\sqrt{2}} \left[\begin{array}{c} \uparrow \text{---} \downarrow \\ \downarrow \text{---} \uparrow \end{array} \right]$$

High temperature superconductivity (1986)

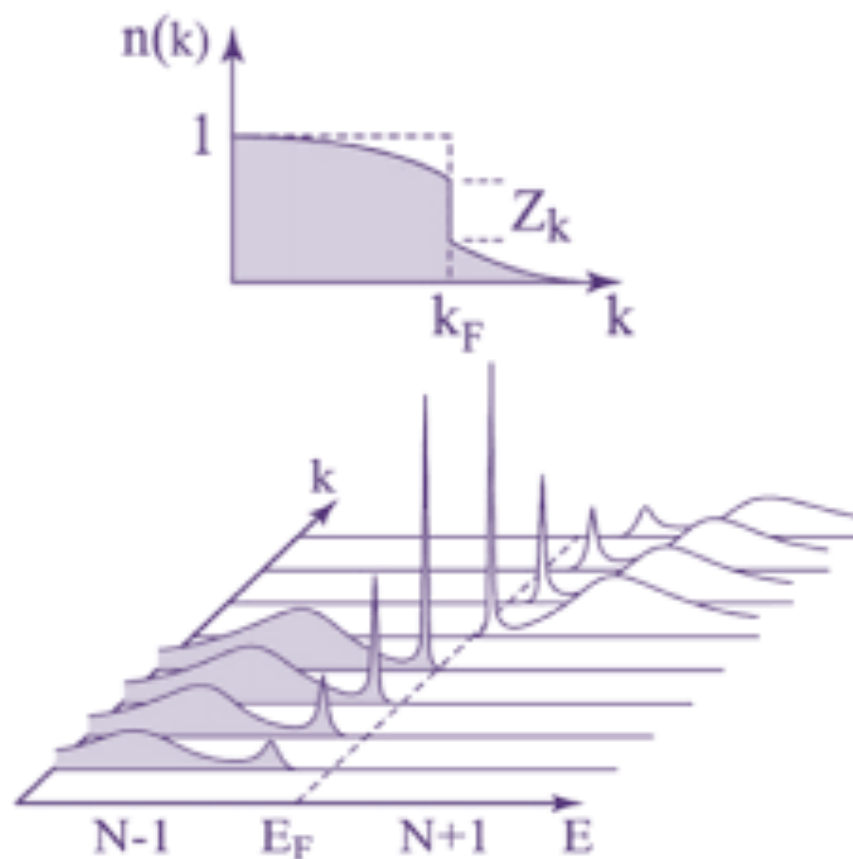


The idea is to view Mott insulator (QSL) as the parent state of high-temperature superconductor. In the QSL, there are preformed Cooper pairs. Doping it allows to Cooper pairs to condense and lead to superconductivity.

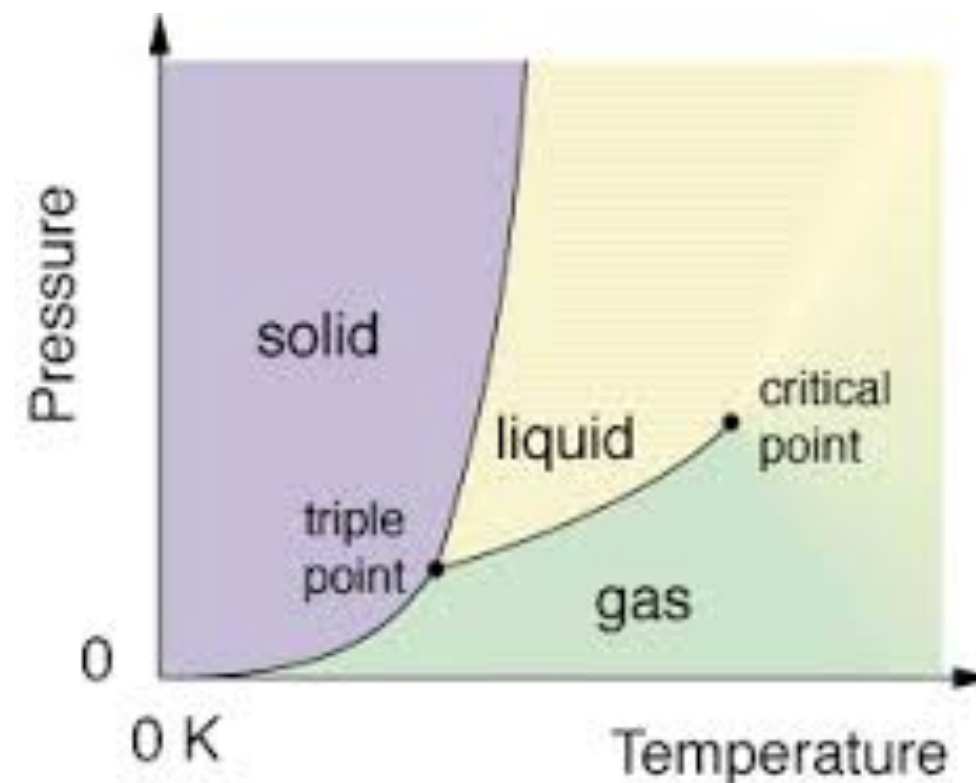


Landau

Two milestones of 20th century condensed matter physics



Landau Fermi liquid theory



Landau symmetry breaking theory

These two paradigms break down after the discovery of fractional quantum Hall effect in 1980s.

Quantum spin liquid

- Quantum spin liquid is a new quantum phase of matter, and cannot be characterized by Landau symmetry breaking, instead by emergent gauge structure and deconfined fractionalized excitations.
- QSL, its existence, is very clear, at least at the level of theory.
 - Exactly solvable model with QSL ground state: e.g. Kitaev model and extension.
 - Classification of QSLs: many distinct symmetry enriched QSLs (XG Wen etc).
 - Numerical solutions: DMRG, QMC, exact diagonalization, etc.

QSL is **robust** against any local perturbation. So it should exist in Nature !

QSL: existing experiments

- 2D triangular and Kagome lattice
organics: κ -(BEDT-TTF) $_2$ Cu $_2$ (CN) $_3$, EtMe $_3$ Sb[Pd(dmit) $_2$] $_2$, κ -H $_3$ (Cat-EDT-TTF) $_2$
herbertsmithite (ZnCu $_3$ (OH) $_6$ Cl $_2$), Ba $_3$ NiSb $_2$ O $_9$, Ba $_3$ CuSb $_2$ O $_9$, LiZn $_2$ Mo $_3$ O $_8$, ZnCu $_3$ (OH) $_6$ Cl $_2$
volborthite (Cu $_3$ V $_2$ O $_7$ (OH) $_2$), BaCu $_3$ V $_2$ O $_3$ (OH) $_2$, [NH $_4$] $_2$ [C $_7$ H $_{14}$ N][V $_7$ O $_6$ F $_{18}$], Na $_2$ IrO $_3$, CsCu $_2$ Cl $_4$, CsCu $_2$ Br $_4$, NiGa $_2$ S $_4$, He-3 layers on graphite, etc
- 3D pyrochlore, hyperkagome, FCC lattice, diamond lattice, etc
Na $_4$ Ir $_3$ O $_8$, IrO $_2$, Ba $_2$ YMoO $_6$, Yb $_2$ Ti $_2$ O $_7$, Pr $_2$ Zr $_2$ O $_7$, Pr $_2$ Sn $_2$ O $_7$, Tb $_2$ Ti $_2$ O $_7$, Nd $_2$ Zr $_2$ O $_7$, FeSc $_2$ S $_4$, etc
- Ultracold atom and molecules on optical lattices: temperature is too high now.

Some candidate materials have already been ruled out.

Not being a QSL does not necessarily mean the physics is not interesting !

- Spinon Fermi surface $U(1)$ quantum spin liquid

Any guiding rule to find QSL? Not really.

Frustrated lattice?	Honeycomb Kitaev model.
Frustrated interaction?	We do not really know unless we identify the interaction.
Low dimensionality?	3D lattice also has QSL.
Odd electrons per cell?	Many QSLs have even electrons per cell.



Lieb



Oshikawa



Hastings



Vishwanath

- Hastings-Oshikawa-Lieb-Shultz-Mattis theorem.
- Recent extension to spin-orbit coupled insulators (Watanabe, Po, Vishwanath, Zaletel, PNAS 2016).

A rare-earth triangular lattice quantum spin liquid: **YbMgGaO₄**



Dr. Yuesheng Li

Renmin Univ -> MPI, Germany

This part is in collaboration with experimentalists

Dr. Yuesheng Li (Renmin Univ, Beijing)

Prof. Qingming Zhang (Renmin Univ, Beijing)

Wei Tong (High Magnetic field Lab, Hefei)

Pi Li (High Magnetic field Lab, Hefei)

Juanjuan Liu (Renmin Univ, Beijing)

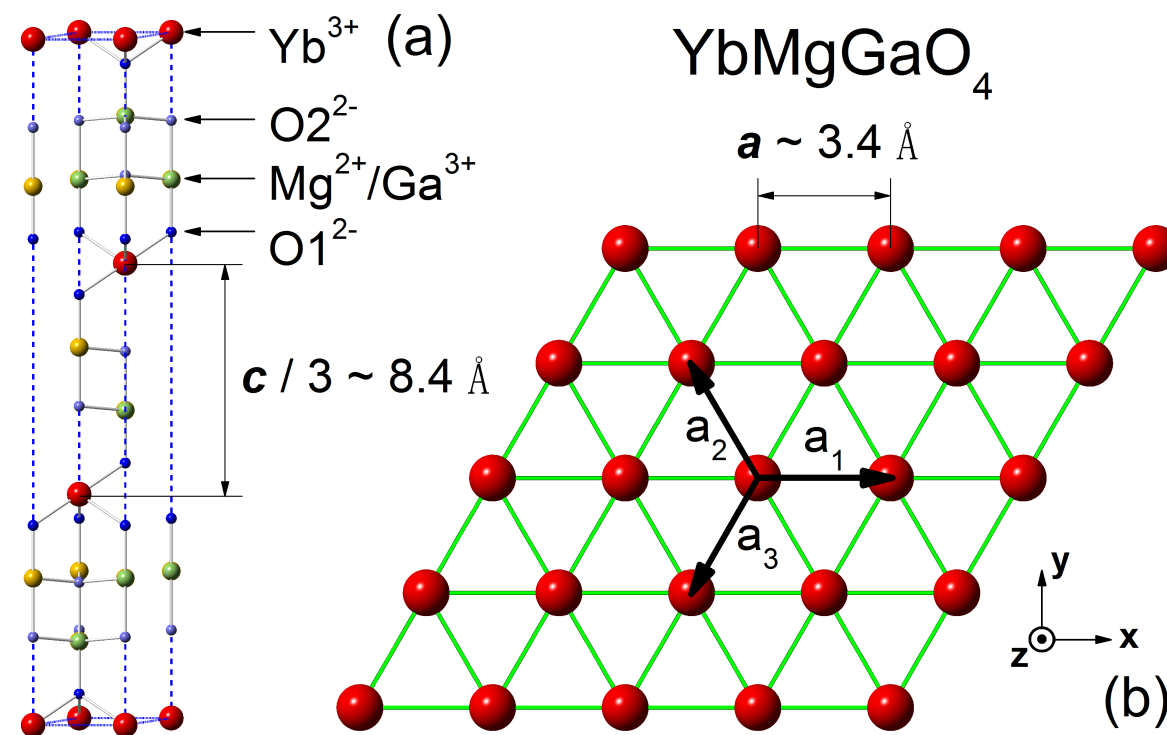
Zhaorong Yang (Institute of Solid-State Physics, Hefei)

Xiaoqun Wang (Renmin, Shanghai Jiaotong)

YS Li, **GC***,, QM Zhang*

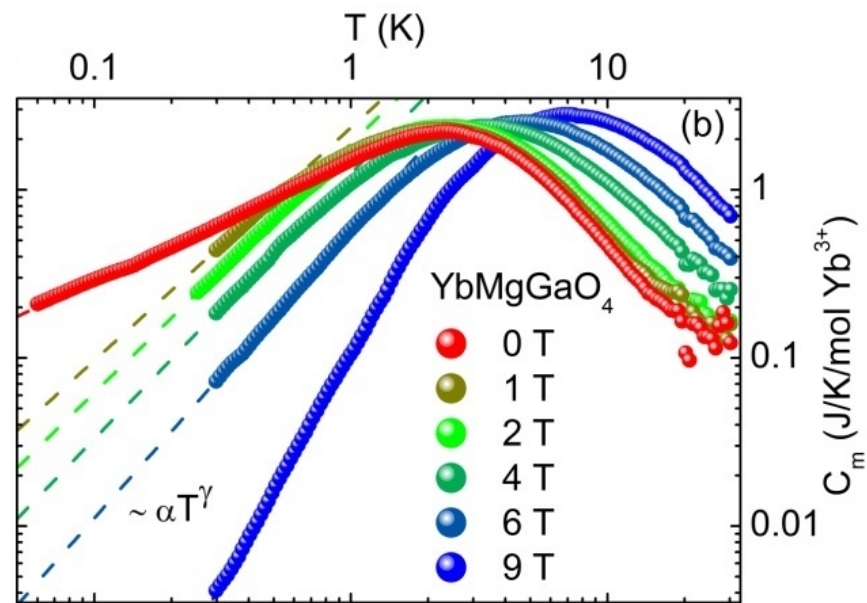
PhysRevLett 2015

A rare-earth triangular lattice quantum spin liquid: **YbMgGaO₄**

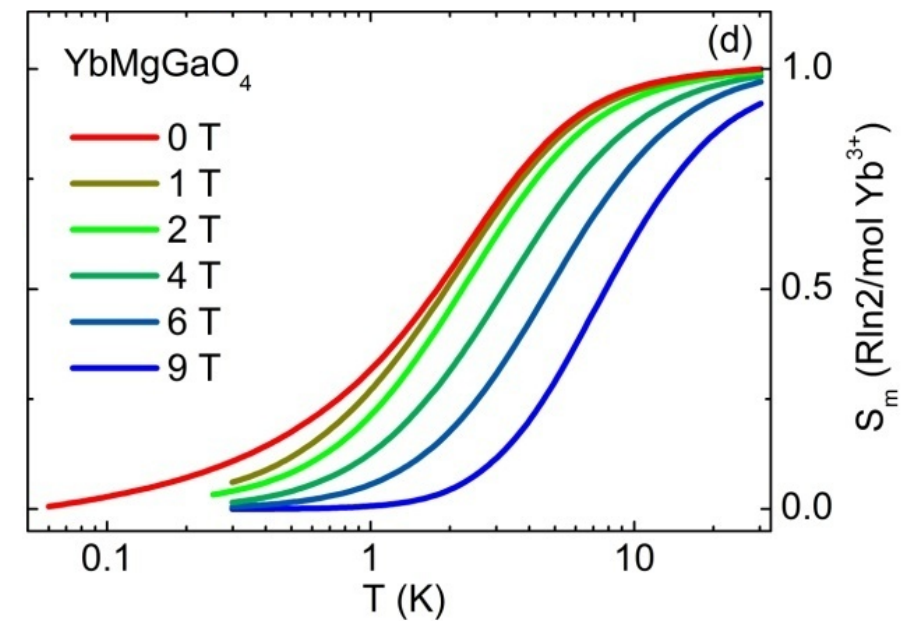


- This is the **first** strong spin-orbit coupled QSL with odd number of electrons and effective spin-1/2.
- It is the **first** clear observation of $T^{2/3}$ heat capacity.
- We understand the microscopic Hamiltonian and the physical mechanism.

YbMgGaO₄



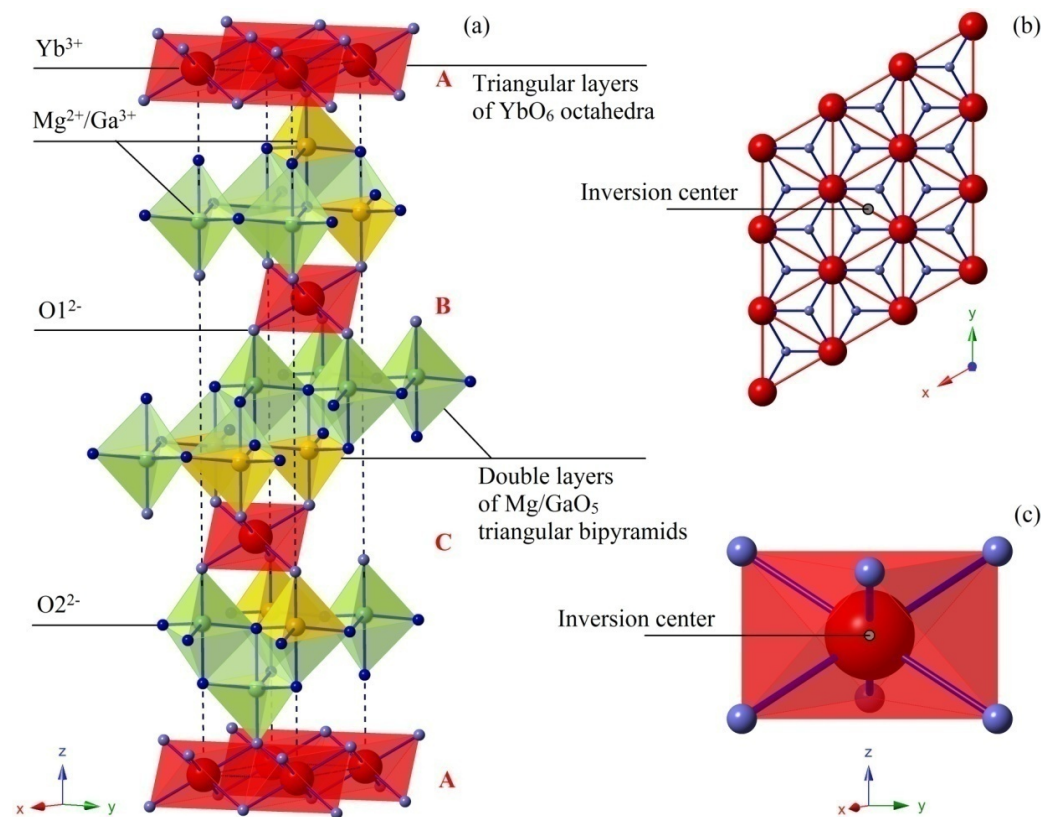
- observation of $T^{2/3}$ heat capacity



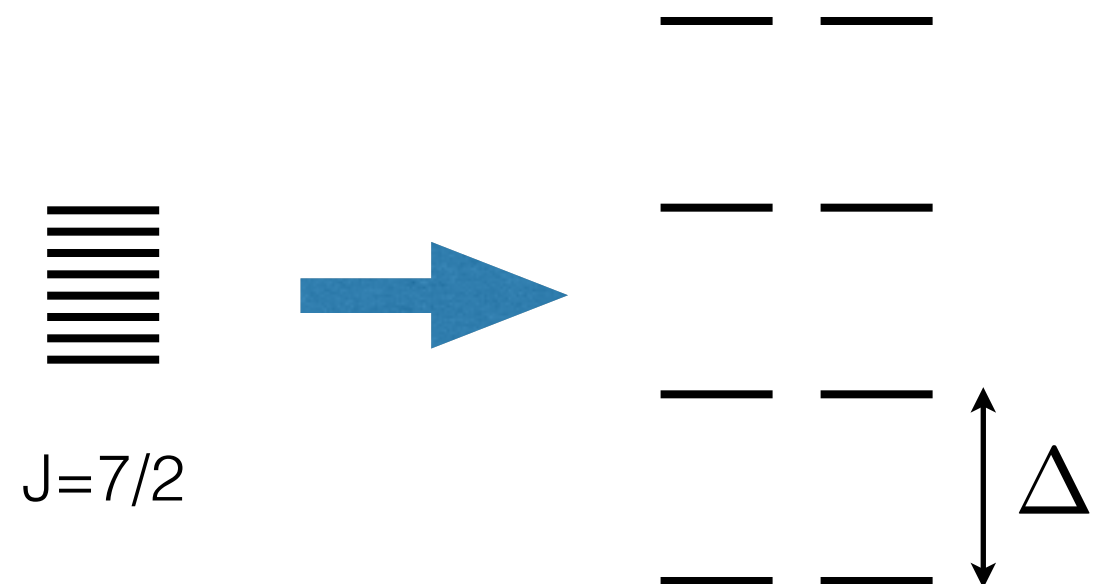
- Entropy: effective spin-1/2 local moments

My proposal for ground state: spinon Fermi surface U(1) QSL.

Microscopics



Yb^{3+} ion: $4f^{13}$ has $J=7/2$ due to SOC.



At $T \ll \Delta$, the only active DOF is the ground state doublet that gives rise to an effective spin-1/2.

Can this kind of system support a QSL ground state? Yes.

Filling constraints for spin-orbit coupled insulators in symmorphic and non-symmorphic crystals

Haruki Watanabe,¹ Hoi Chun Po,¹ Ashvin Vishwanath,^{1,2} and Michael P. Zaletel³

May 2015

and a crystalline lattice or a magnetic field. Mott insulators are a particularly interesting class, with an odd number of electrons in each unit cell. Their low energy physics is captured by a spin model with an odd number of $S = 1/2$ moments in the unit cell. A powerful result due to Lieb, Schultz, and Mattis in 1D¹, later extended to higher dimensions by Hastings and Oshikawa^{2,3}, holds that if all symmetries remain unbroken, the ground state must be ‘exotic’ - such as a Luttinger liquid in 1D, or a quantum spin liquid in higher dimensions, with fractional ‘spinon’ excitations. These exotic states cannot be represented as simple product states, as a consequence of long ranged quantum entanglement. This general re-

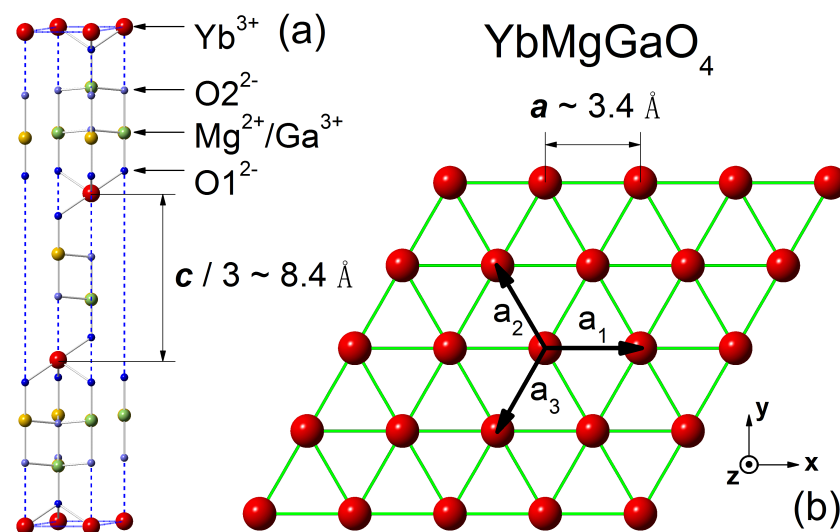
tirely different theoretical approaches are needed. We argue that if a spin-orbit coupled insulator at odd filling is time-reversal symmetric, its ground state must, in a precise sense, be exotic. We introduce two theoreti-

“this kind of system” means effective spin-1/2, spin-orbit coupling, odd number of electron per cell.

as you may know, there are many theoretical works trying to constrain the possible states from being existing.

What is the physical origin of the QSL?

4f electron is very localized, and dipolar interactions weak.



PhysRevLett, 2015

$$\mathcal{H} = \sum_{\langle ij \rangle} [J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) - \frac{iJ_{z\pm}}{2} (\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + \langle i \leftrightarrow j \rangle)], \quad (1)$$

where $S_i^{\pm} = S_i^x \pm iS_i^y$, and the phase factor $\gamma_{ij} = 1, e^{i2\pi/3}, e^{-i2\pi/3}$ for the bond ij along the $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ direction (see Fig. 1), respectively. This generic Hamil-

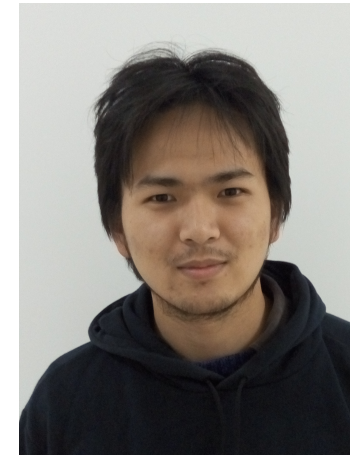
The spin-1/2 XXZ model supports conventional order.

(Yamamoto, etc, PRL 2014)

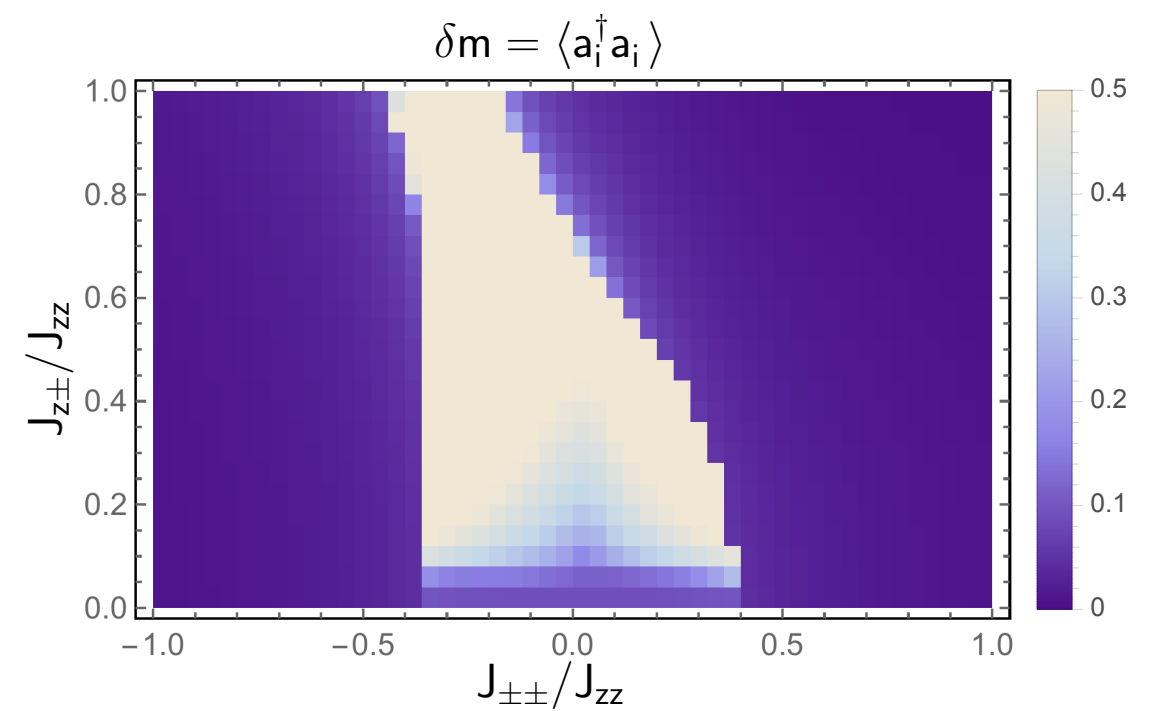
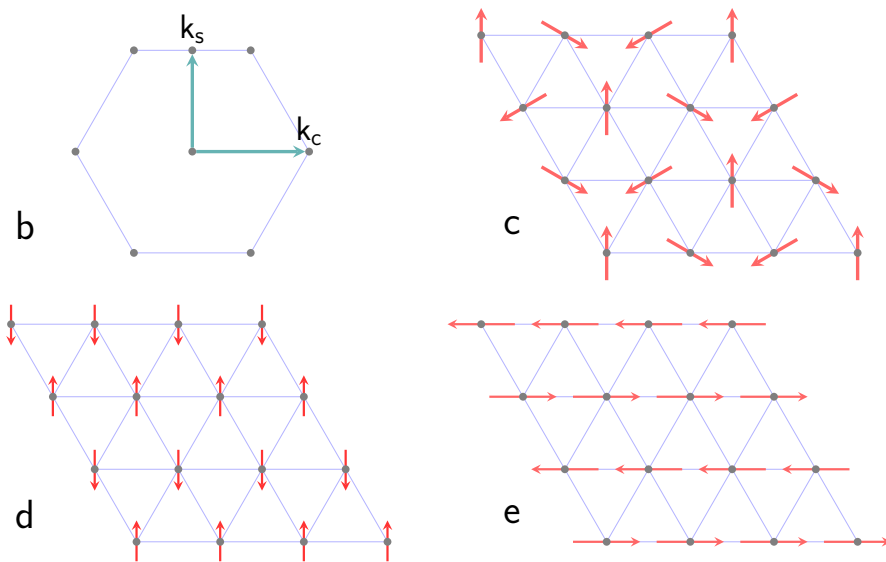
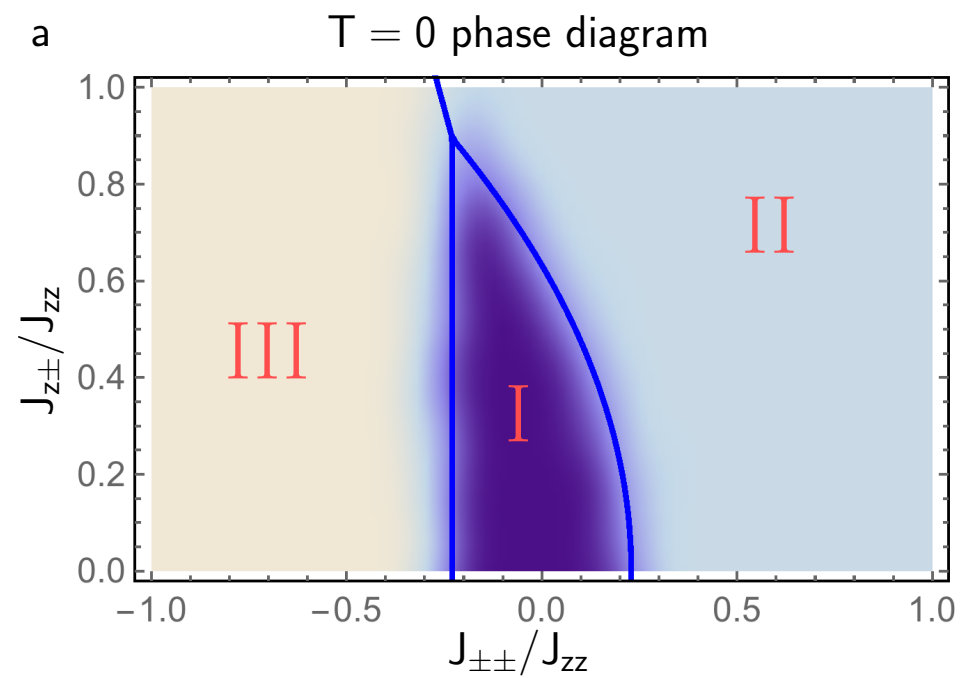
This will makes George happy.

This model

Anisotropic spin interaction
could potentially stabilize QSL.



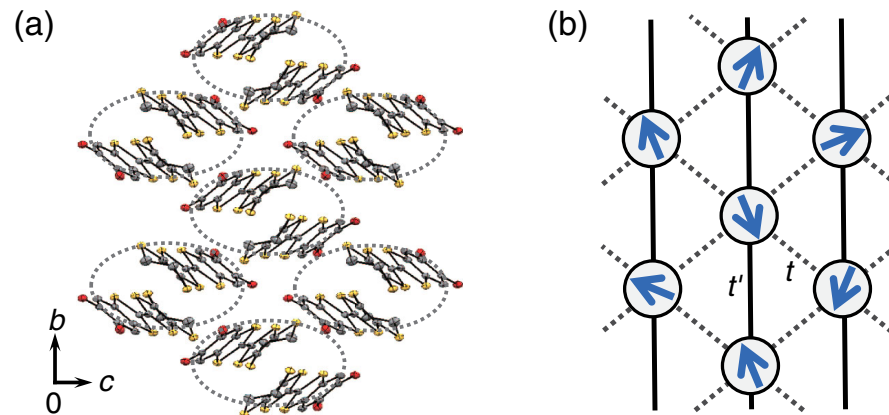
Yao-Dong Li
Dept of Computer Sciences
Fudan University



Spinon Fermi surface U(1) QSL in organic magnets?

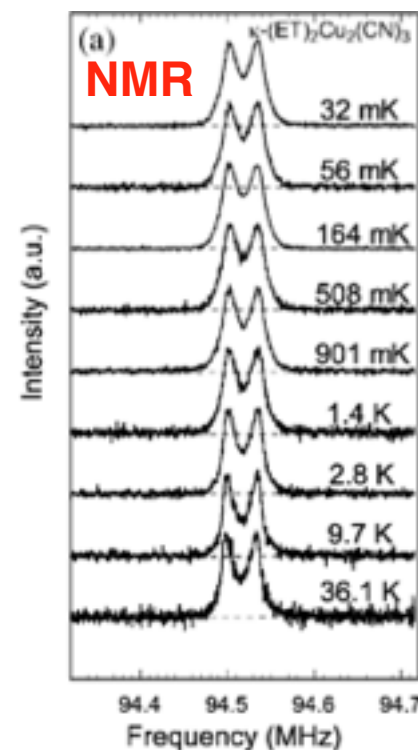
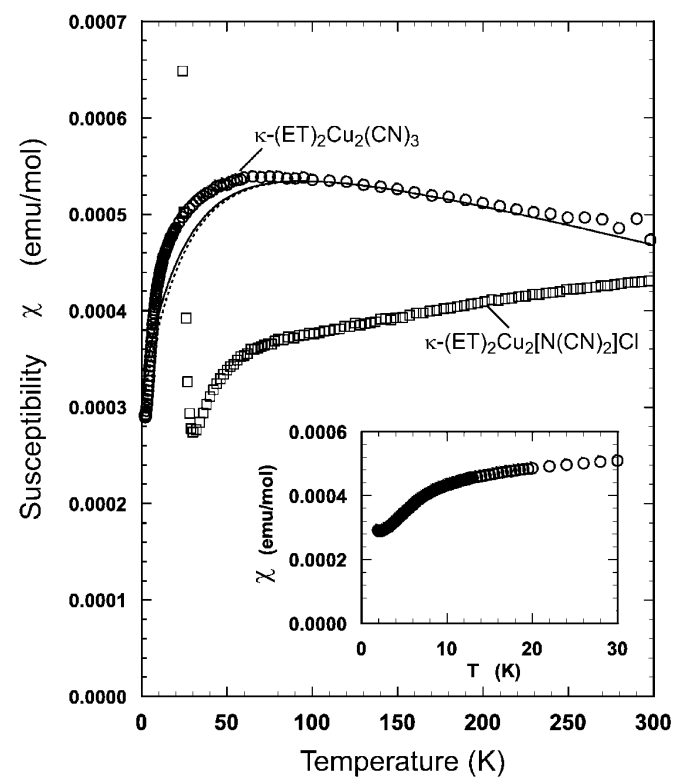


Kanoda



κ -(BEDT-TTF)₂Cu₂(CN)₃,
EtMe₃Sb[Pd(dmit)₂]₂,
 κ -H₃(Cat-EDT-TTF)₂

a new one!



- * No magnetic order down to 32mK
- * Constant spin susceptibility at zero temperature

Other experiments: transport,
heat capacity, optical absorption, etc,
Unfortunately, **no neutron scattering** so far.

- Theoretical understanding: expected phase diagram

$$H = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + h.c. + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Sung-Sik Lee

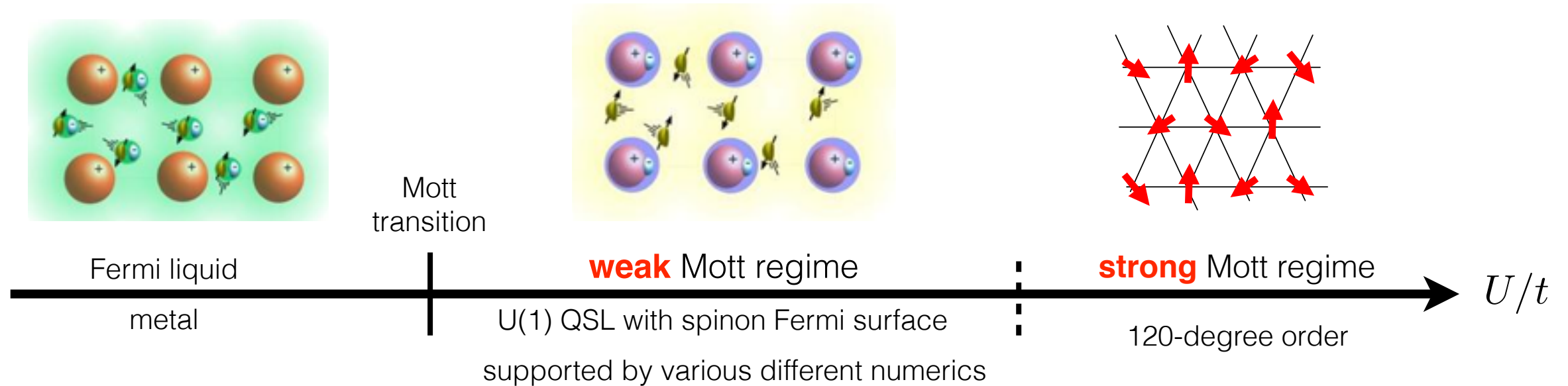


T Senthil



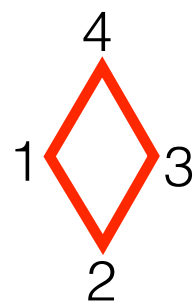
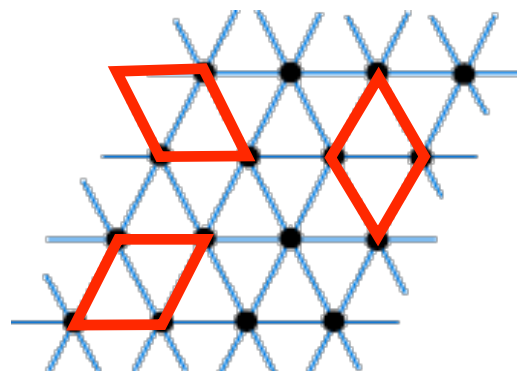
Patrick Lee

Senthil's cartoon



- Physical mechanism** for weak Mott insulator spin liquids: perturbation in t/U

$$H_{\text{pert}} = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + K \sum_{1234} (P_{1234} + P_{1234}^{-1}) + \dots$$



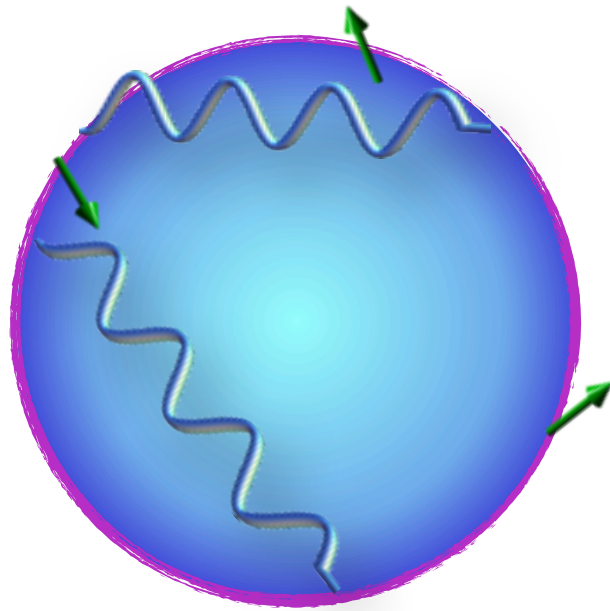
4-site ring exchange

$$(\mathbf{S}_1 \cdot \mathbf{S}_2)(\mathbf{S}_3 \cdot \mathbf{S}_4) + (\mathbf{S}_1 \cdot \mathbf{S}_4)(\mathbf{S}_2 \cdot \mathbf{S}_3) - (\mathbf{S}_1 \cdot \mathbf{S}_3)(\mathbf{S}_2 \cdot \mathbf{S}_4)$$



Motrunich

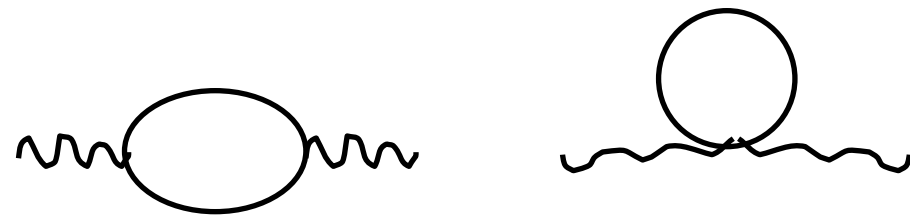
Low energy property of spinon Fermi surface U(1) QSL: spinon non-Fermi liquid



Spinon Fermi surface coupled
with dynamical U(1) gauge field:
instanton event is suppressed.

dual to extremal/charged black hole?

$$S = \int d^3x \left[\Psi_j^* (\partial_0 - ia_0 - \mu_F) \Psi_j + \frac{1}{2m} \Psi_j^* (-i \nabla - \mathbf{a})^2 \Psi_j + \frac{1}{4g^2} f_{\mu\nu} f_{\mu\nu} \right].$$



gauge photon is overly Landau-damped.

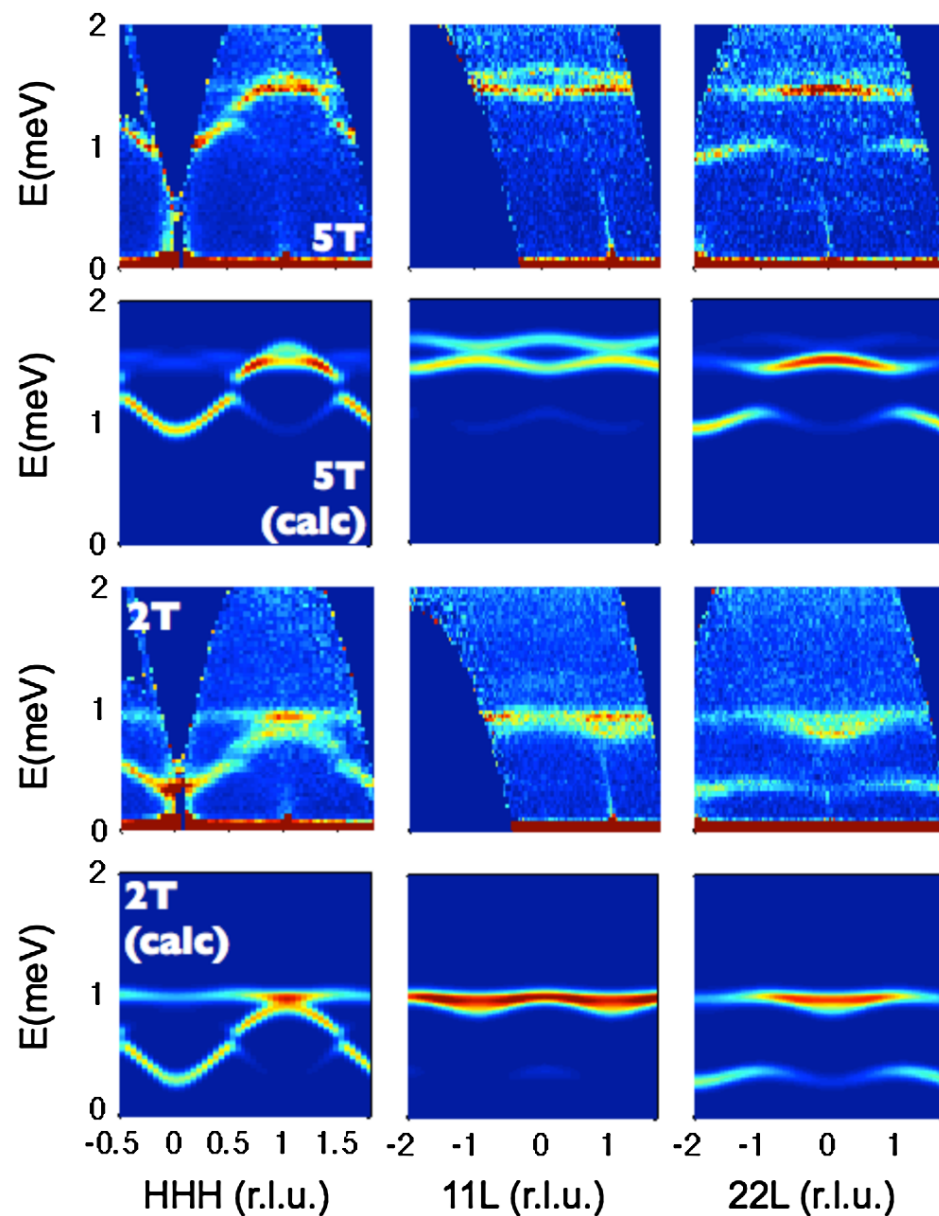


$$Re\Sigma \sim Im\Sigma \sim \omega^{2/3}$$

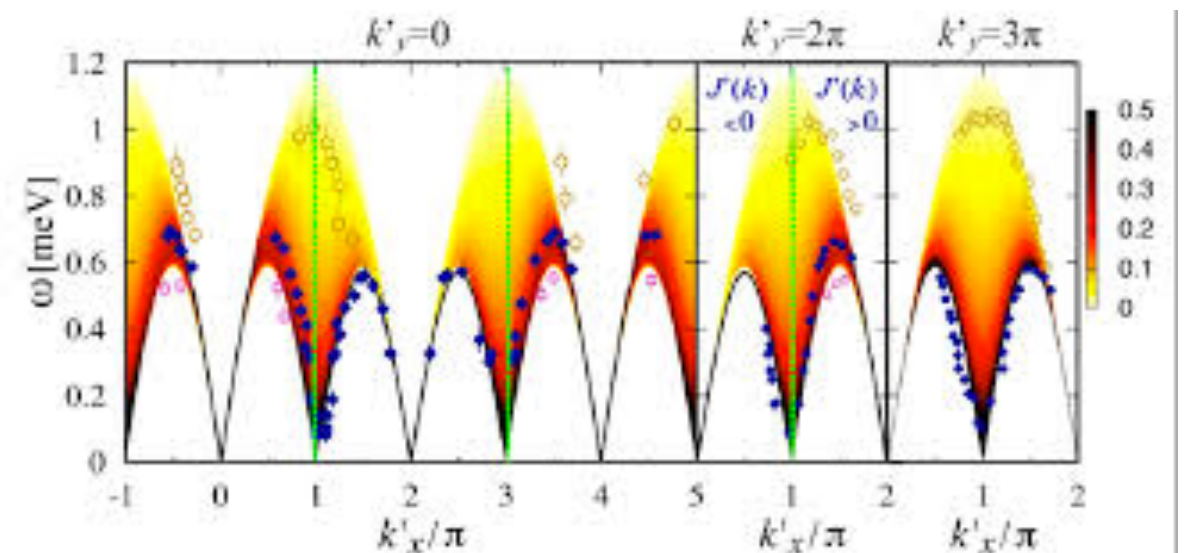
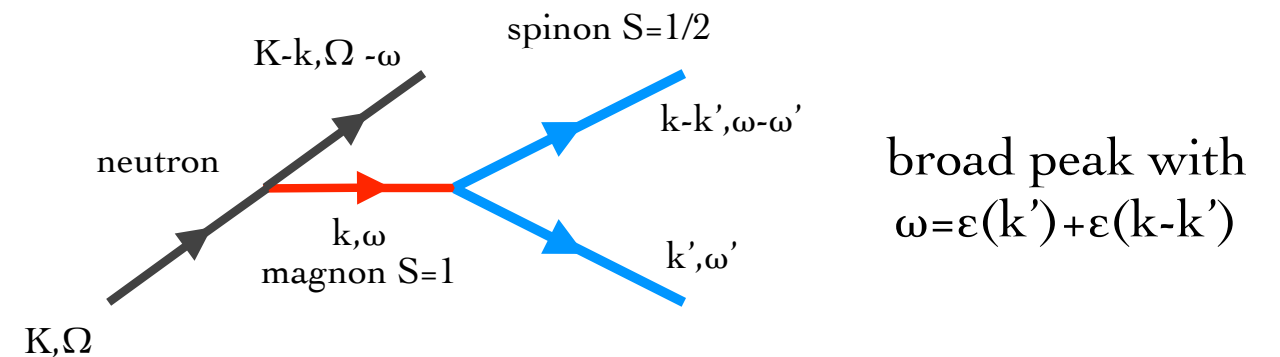
Hermele et al., PRB 70, 214437 (04)

Sung-Sik Lee, PRB 78, 085129(08).

Spin wave vs (fractionalized) spinon continuum

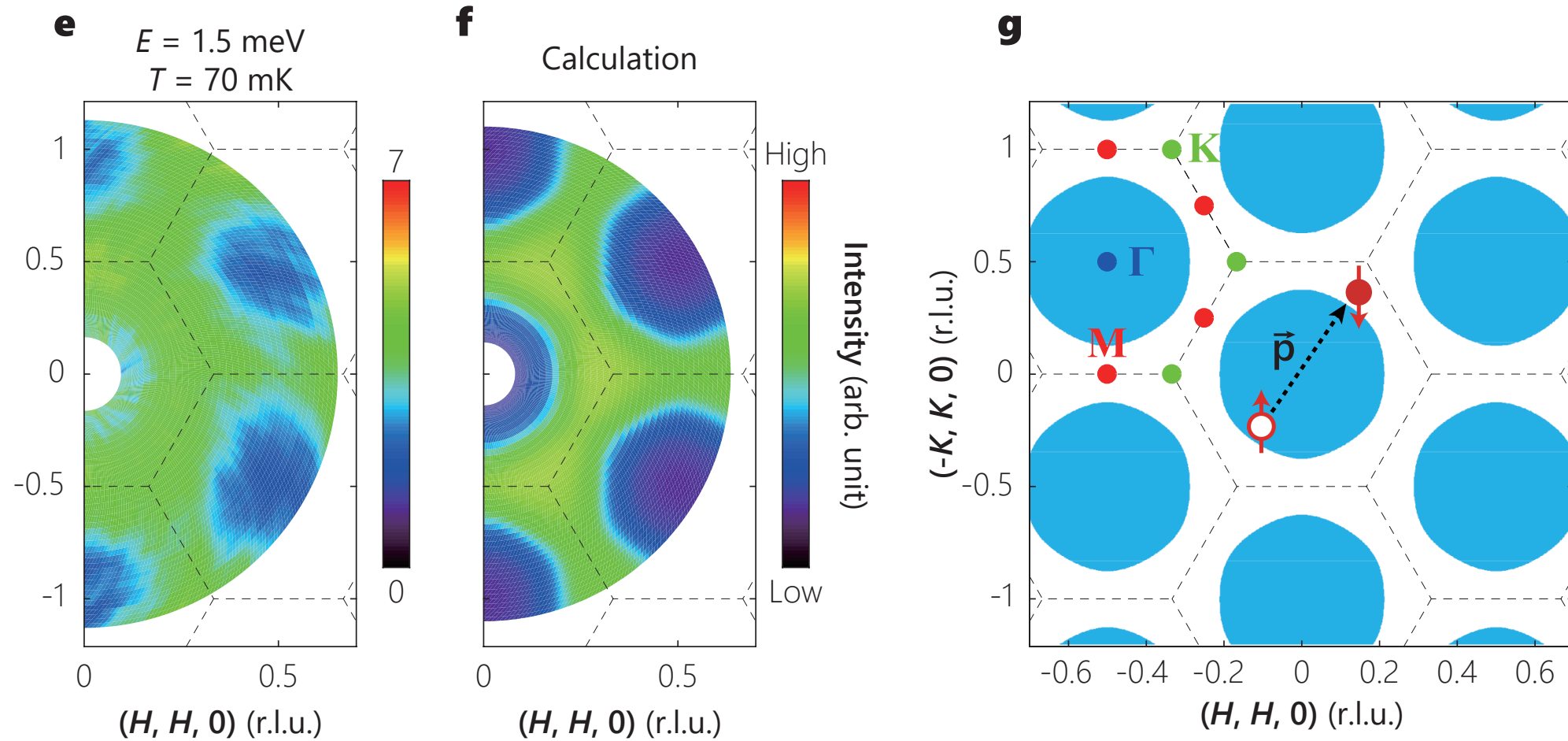


spin wave in $\text{Yb}_2\text{Ti}_2\text{O}_7$
L Savary, et al, PRX 2011

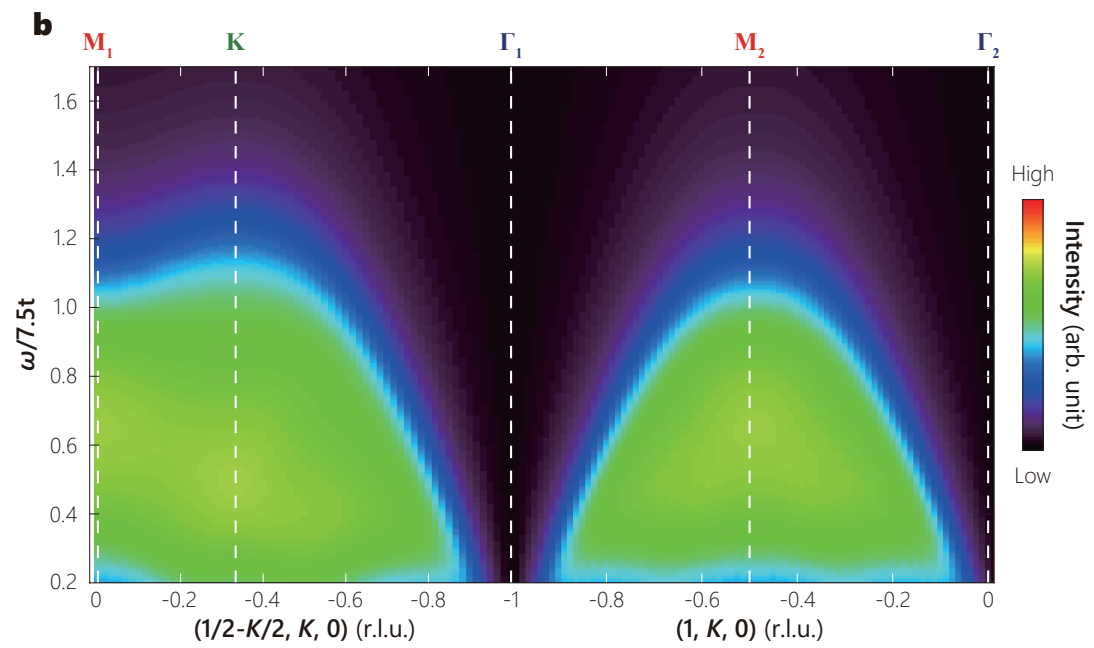
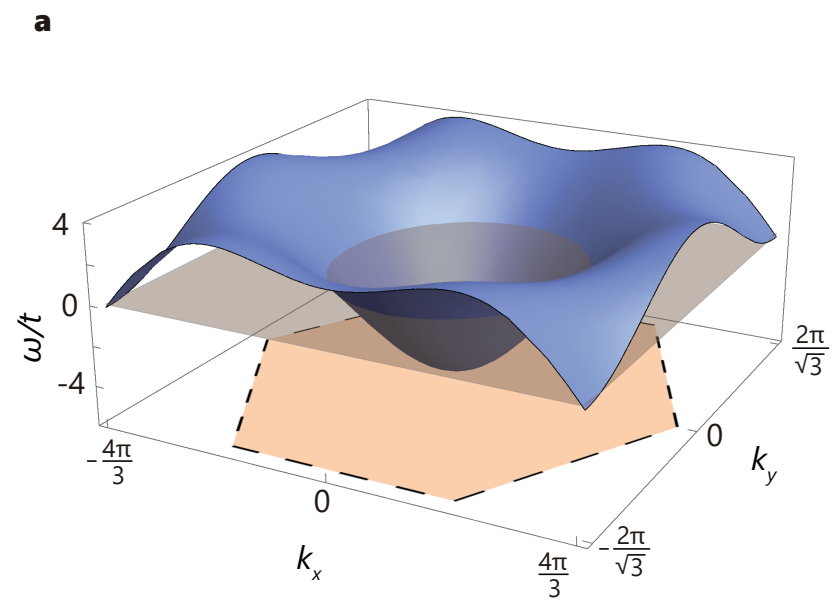
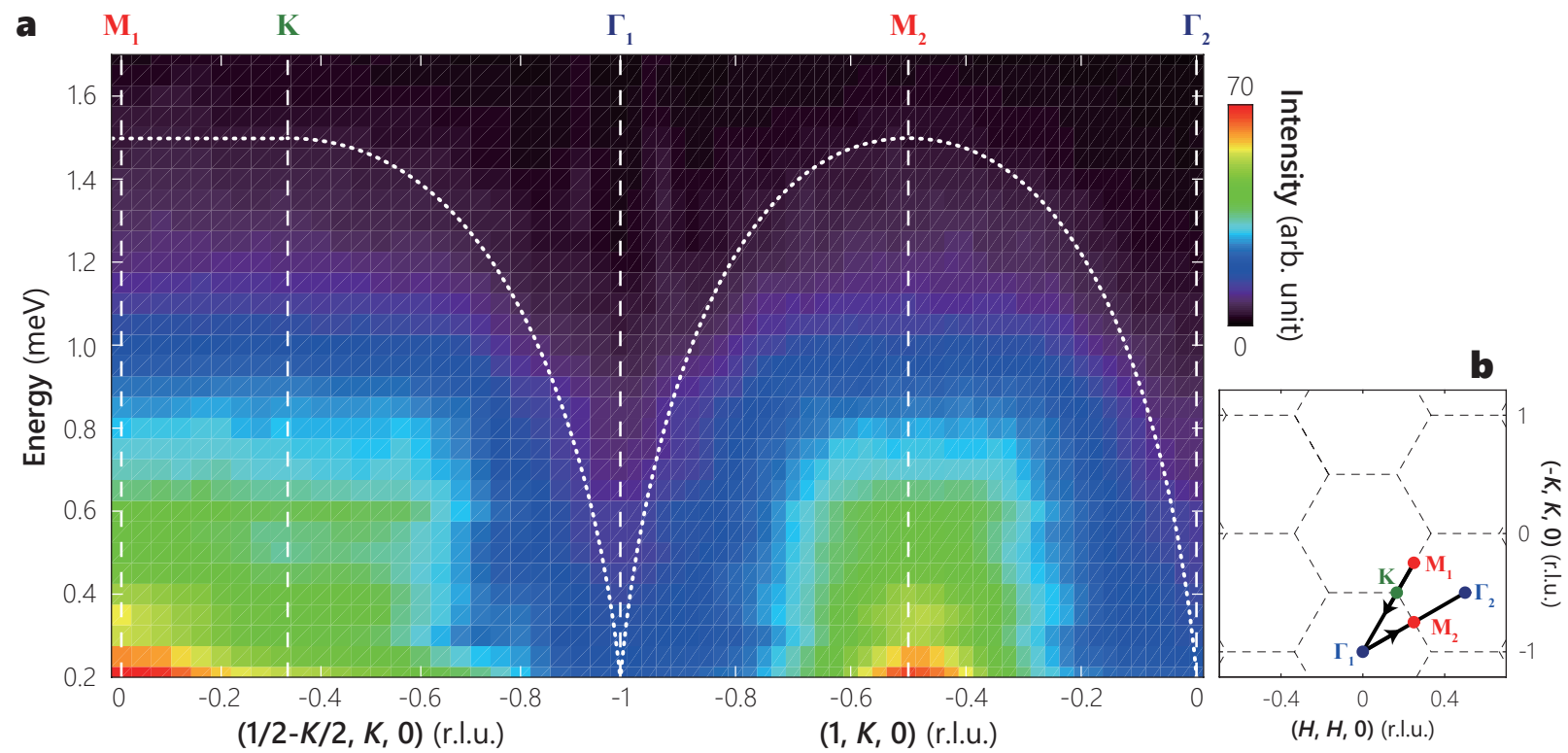


spinon continuum in Cs_2CuCl_4
Masanori, etc NatPhys 2009
but these are **1d spinons** !

Huge spinon continuum at all energies



Yao Shen, ...GC, Jun Zhao arxiv 2016

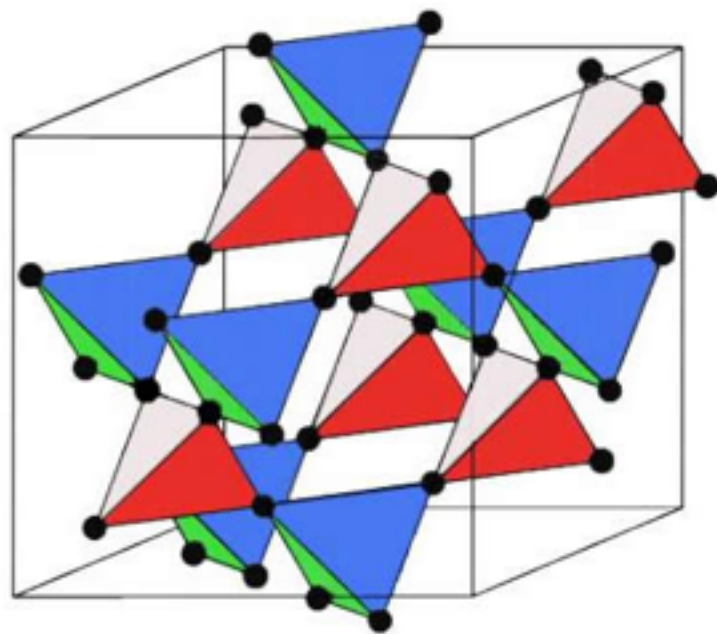


Summary

1. QSL is a field that bridges the fundamental ideas with the frontier experiments, it provides exciting opportunities for both theorists and experimentalists.
2. Rare-earth triangular lattice quantum spin liquid: **YbMgGaO₄**
 - To our best knowledge, this is the **first strong spin-orbit coupled** quantum spin liquid candidate with odd number of electrons per unit cell and effective spin-1/2 moment.

- Octupolar $U(1)$ quantum spin liquid
of quantum spin ice

Rare-earth pyrochlores



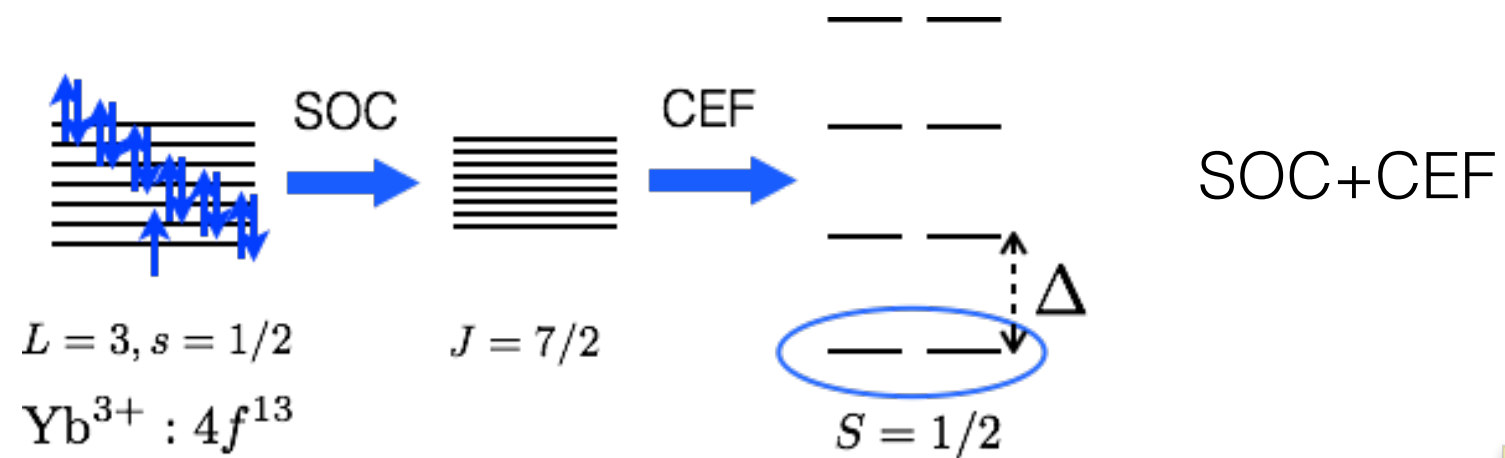
Rare Earth Elements

H									He
Li	Be								Ne
Na	Mg								Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Xe
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Rn
Fr	Ra	Ac-Lr	Rf	Db	Sg	Bh	Hs	Mt	
Lanthanides									
		La	Ce	Pr	Nd	Pm	Sm	Eu	
Actinides									
		Ac	Th	Pa	U	Np	Pu	Am	

Over years, there are a lot of activity in spin ice system.

the rare earth moment form a pyrochlore lattice, which is corner sharing tetrahedron in 3d

Rare-earth local moments: a **crude** classification



Kramers' doublet: R^{3+} with **odd** number of electrons

lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93
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Non-Kramers' doublet / singlet: R^{3+} with **even** number of electrons

lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
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in a rare earth ion site, the f orbitals form a spin-orbit entangled state with total angular momentum J , and then CEF splits this into multiple doublet, or singlet states. The excited state is much higher in energy than the energy scale of the CEF splitting, so one can simply focus on the ground state doublet, which is modeled as an effective spin-1/2 DOF.

Crudely speaking, the rare earth ions can be classified into Kramers' doublet and non-Kramers' doublet / singlet. Rare earth ions with odd number of f electrons form Kramers' doublet, even number of f electrons form non-Kramers' doublet / singlet.

if one google search, one can find a lot of nice work by these distinguished theoretical physicists.

Spin ice (Ising) limit

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z + \dots$$

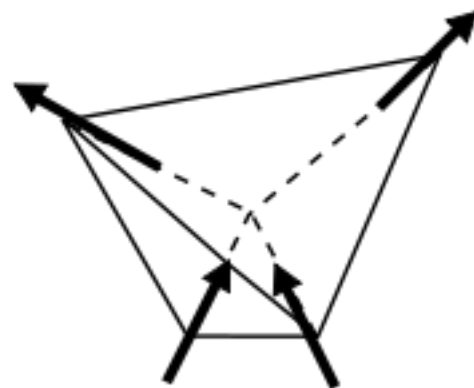
Over years, there are a lot of activity in spin ice system.

spin ice is realized in rare earth pyrochlore systems, when the ising part of the exchange is AFM and dominant.

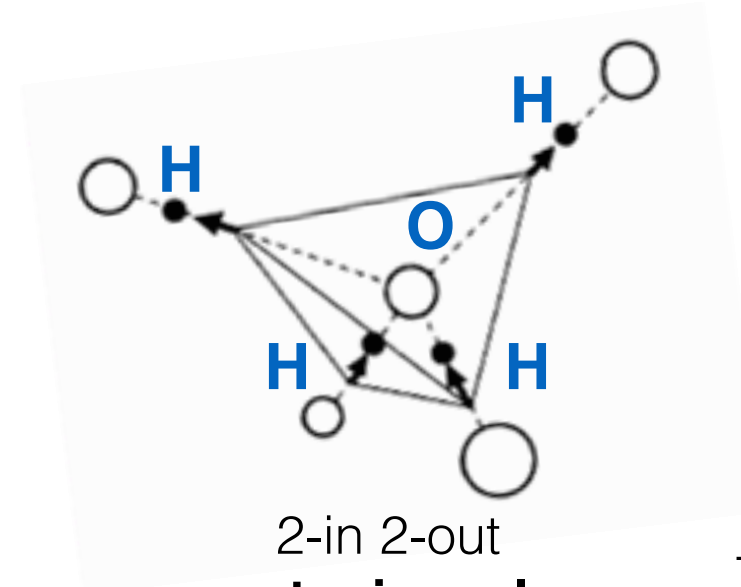
if one google search, one can find a lot of nice work by these distinguished theoretical physicists.

The interaction between the ising is AFM, it favor 2 spin in 2 spin out of the tetrahedra. This is the 2-in 2-out spin ice rule.

Beucase of the analog relation with H position in water ice, each O has 4 H near it, 2 are close, 2 are further.



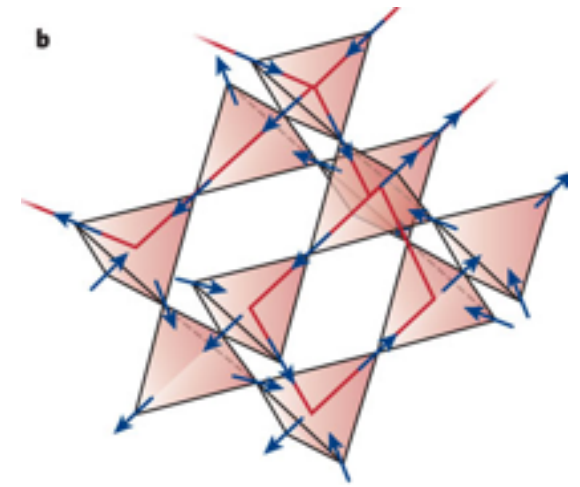
2-in 2-out
spin ice rule



2-in 2-out
water ice rule

from wiki

Classical spin ice



- The “2-in 2-out” states are extensively degenerate.
- At temperature $T < J_{zz}$, the system **thermally** fluctuates within the ice manifold, leading to classical spin ice and interesting experimental discoveries.



1. the 2-in 2-out spin ice states are extensively degenerate. In a tetrahedron, one can have 2-in 2-out or 2-out 2-in states.
2. at $T \ll J_{zz}$, the system fluctuates within the ice manifold, leading to classical spin ice and interesting experimental consequences. Many of these are discussed in nature and science.

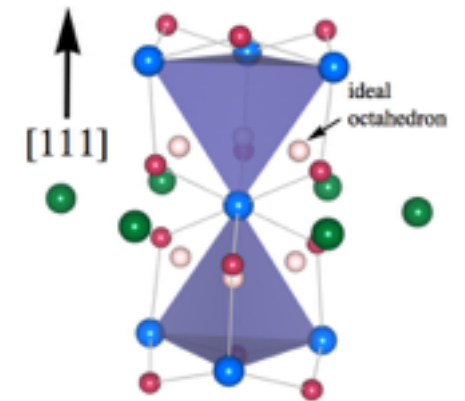
Pinch points in spin correlation

Dipole-octupole doublet

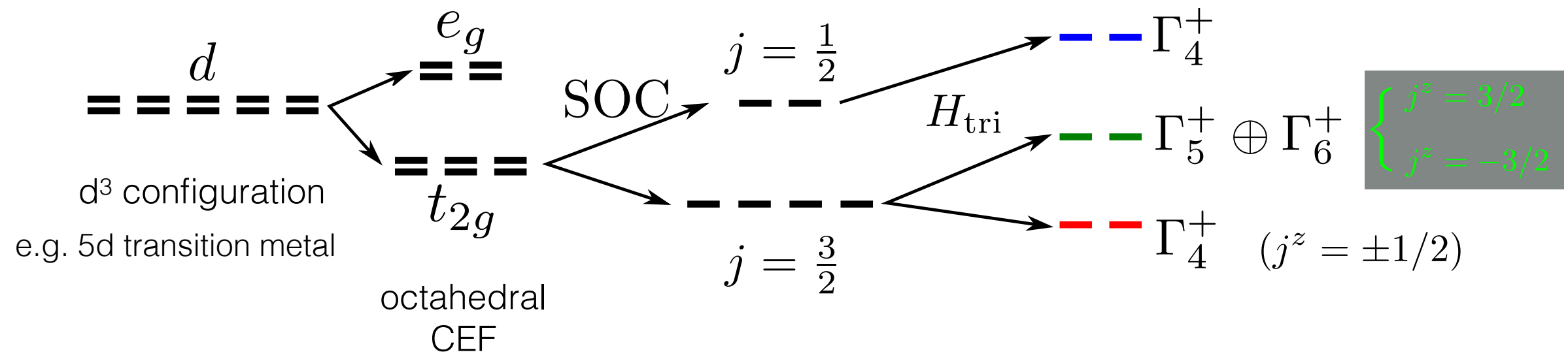
The early classification of local moments is a bit crude !
One should carefully examine the wavefunction of the local doublet.

what is dipole-octupole doublet, the early classification is a bit crude,
one should carefully examine the wavefunction of the local doublet .

Local physics: start with t_{2g} electrons



- Local moments on pyrochlore lattice: effective spin-1/2



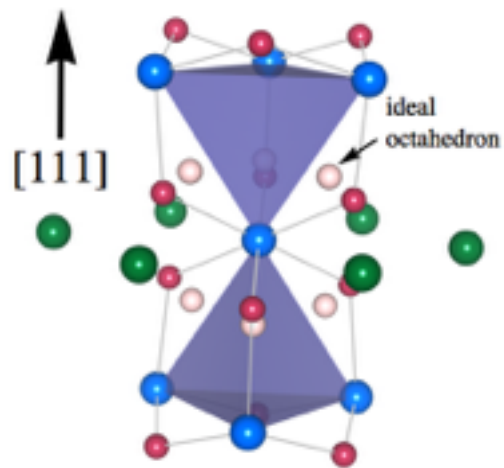
d electrons under D_{3d} point group crystal field

for my convenience, let me start with t_{2g} electrons. Let's consider 5d electron in octahedral environment.

add $J_z = \pm 1/2$

- Why is this Kramers doublet so special ?

ONE-dimensional representations of the point group



$$R(2\pi/3)|J^z = \pm 3/2\rangle = -|J^z = \pm 3/2\rangle$$

$$R(2\pi/3) \equiv e^{-i\frac{2\pi}{3}J^z} = e^{-i\frac{2\pi}{3} \times (\pm \frac{3}{2})} = e^{\mp i\pi} = -1$$

$$|J^z = +3/2\rangle \xrightarrow{\text{time reversal}} |J^z = -3/2\rangle$$

why special,

they are one dimensional rep of point group.

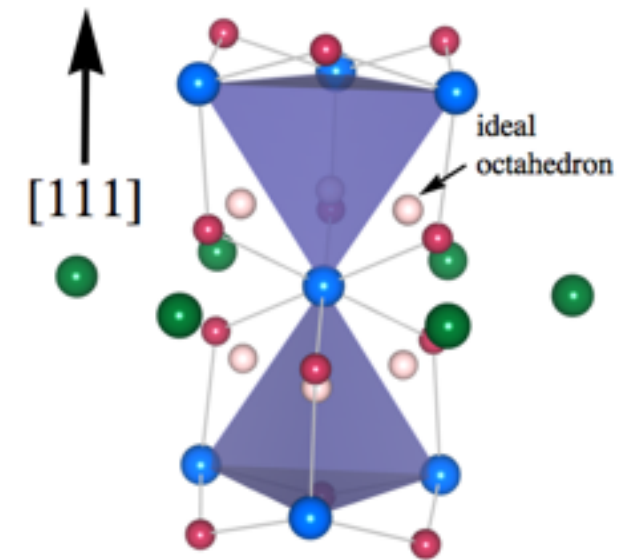
in particular, if you look at the 3-fold rotation operation, under this rotation each state stay invariant, except a minus sign. they do not transform into each other under the point group transformation.

simple algebra

this is very different from 2-dim irreps where these two states of the doublet are mixed.

These two states are degenerate under time reversal, deg protected by time reversal.

More generally, ...



- Also applies to 4f electron moments on pyrochlore

$$J = \frac{3}{2}, \frac{9}{2}, \frac{15}{2}, \dots$$

with the local crystal field Hamiltonian

$$H_{\text{cf}} = 3B_2^0(J^z)^2 + \dots \quad \text{if } B_2^0 < 0$$

more generally, it applies to xxxx.
if the local crystal field hamiltanian is
easy axis like.

e.g. local doublet wavefunction of Dy^{3+} ($J = \frac{15}{2}$) in $\text{Dy}_2\text{Ti}_2\text{O}_7$

$$|\phi_0^\pm\rangle = 0.981|\pm\frac{15}{2}\rangle \pm 0.190|\pm\frac{9}{2}\rangle - 0.022|\pm\frac{3}{2}\rangle \mp 0.037|\mp\frac{3}{2}\rangle + 0.005|\mp\frac{9}{2}\rangle \pm 0.001|\mp\frac{15}{2}\rangle$$

Bertin, etc, J. Phys: cond.mat 2012

Emphasis: what matters is the wavefunction, not the spin value !

- may generally apply to any Kramers' doublets with $J > 1/2$!

e.g, Ce: **Ce₂Sn₂O₇**

PRL **115**, 097202 (2015)

PHYSICAL REVIEW LETTERS

week ending
28 AUGUST 2015

Candidate Quantum Spin Liquid in the Ce³⁺ Pyrochlore Stannate Ce₂Sn₂O₇

Romain Sibille,^{1,*} Elsa Lhotel,² Vladimir Pomjakushin,³ Chris Baines,⁴ Tom Fennell,^{3,†} and Michel Kenzelmann¹

$4f^1$ ion in D_{3d} local symmetry to the susceptibility was realized between $T = 1.8$ and 370 K, and the resulting calculation of the single ion magnetic moment is shown in Fig. 2(c). The wave functions of the ground state Kramers doublet correspond to a linear combination of $m_J = \pm 3/2$ states. The fitted coefficients result in energy levels at $50 \pm$

Ce³⁺ ($4f^1$, 2F)

$$J = \frac{5}{2}$$

the wavefunction should be a combination of states with odd integer values of J , e.g. 3/2.

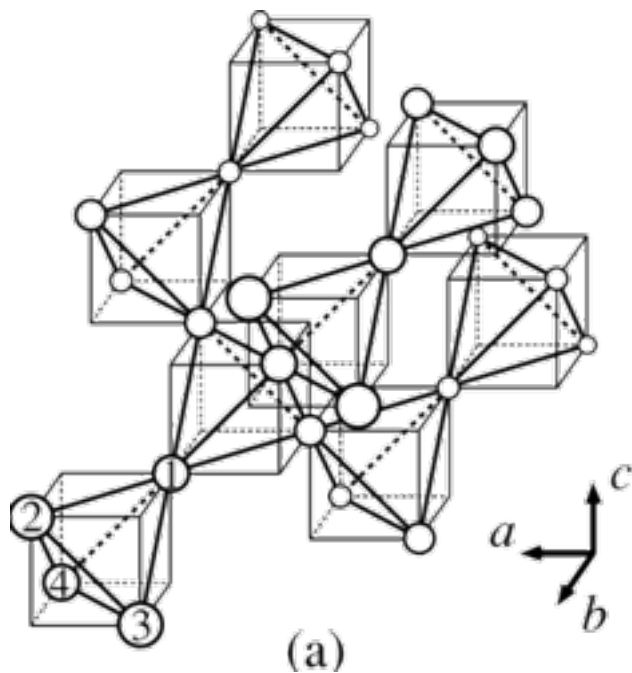
this really because of the fact that the ground states are one-dimensional, even in the YbTiO₂ system, such as the excited doublets.

Realistic **XYZ model** and **Symmetry Enriched** $U(1)$ topological order

next i will introduce the realistic XYZ model that describe the interaction between these peculiar doublet, and discuss the remarkable physical properties.

Symmetry properties

- Effective spin-1/2 under lattice symmetry Tetra



$$T_d \times \mathcal{I} \times \text{translations} \quad \text{and} \quad T_d =$$

$$\begin{cases} S^z = \frac{1}{2} \left| \frac{3}{2} \right\rangle \left\langle \frac{3}{2} \right| - \frac{1}{2} \left| -\frac{3}{2} \right\rangle \left\langle -\frac{3}{2} \right| \\ S^+ = \left| \frac{3}{2} \right\rangle \left\langle -\frac{3}{2} \right|, \quad S^- = \left| -\frac{3}{2} \right\rangle \left\langle \frac{3}{2} \right| \end{cases}$$

$$C_3 : S^\mu \rightarrow S^\mu$$

$$M : S^{x,z} \rightarrow -S^{x,z}, \quad S^y \rightarrow S^y$$

$$\mathcal{I} : S^\mu \rightarrow S^\mu$$

Important: \mathbf{S}^x and \mathbf{S}^z transform identically (as a dipole),
while \mathbf{S}^y transforms as an octupole moment under *mirror*.

just like what we do for any two-fold degree doublet, one can introduce an effective spin-1/2 to describe this doublet.

because of the peculiar property, the symmetry transformation of the effective spin components are rather interesting.

to simplify my notation, i use $j=3/2$ here

Generic model: XYZ model

with the symmetries, one can write down the generic symmetry allowed Hamiltonian.

even though the local moment results from strong spin-orbit coupling, remarkable, the coupling is uniform spatially.

$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$

Uniform Spatially !

VS

$$H = \sum_{\langle ij \rangle} \{ J_{zz} S_i^z S_j^z - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) + J_{z\pm} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j] \},$$

Anisotropic Spatially !

A small transformation into XYZ model

$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$

this does not look like XYZ,

just because of the spatial uniformity,
one can do a rotation in spin space
without effecting real space,



Rotation around **the y axis**
in the effective spin space

$$H_{\text{XYZ}} = \sum_{\langle ij \rangle} \tilde{J}_z \tilde{S}_i^z \tilde{S}_j^z + \tilde{J}_x \tilde{S}_i^x \tilde{S}_j^x + \tilde{J}_y \tilde{S}_i^y \tilde{S}_j^y$$

XYZ model

XXZ model can lead to U(1) QSL

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z - J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) + \dots$$

Hermele, Fisher, Balents, Moessner, Isakov,



- Pretty much one can add any term to create **quantum** tunneling, as not too large to induce magnetic order, the **ground state** is a U(1) Q

1. But classical spin ice is purely classical and is not a new phase of matter. It is smoothly connected to high temperature paramagnetic phase.

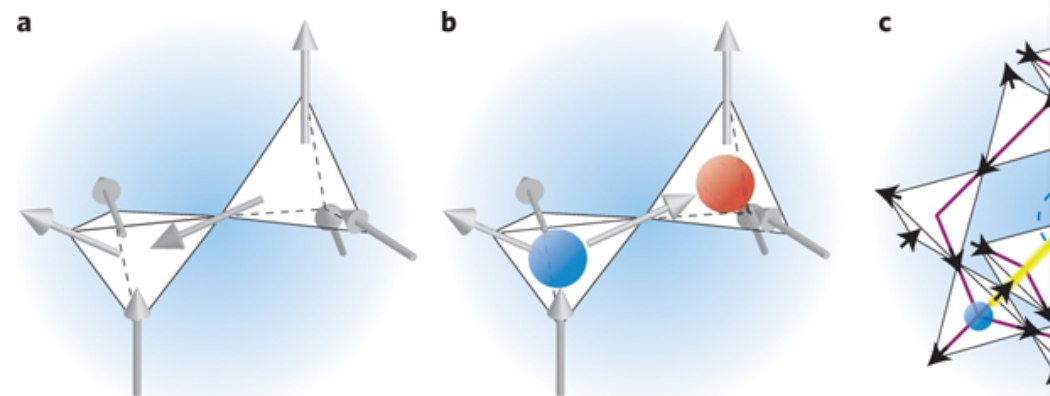
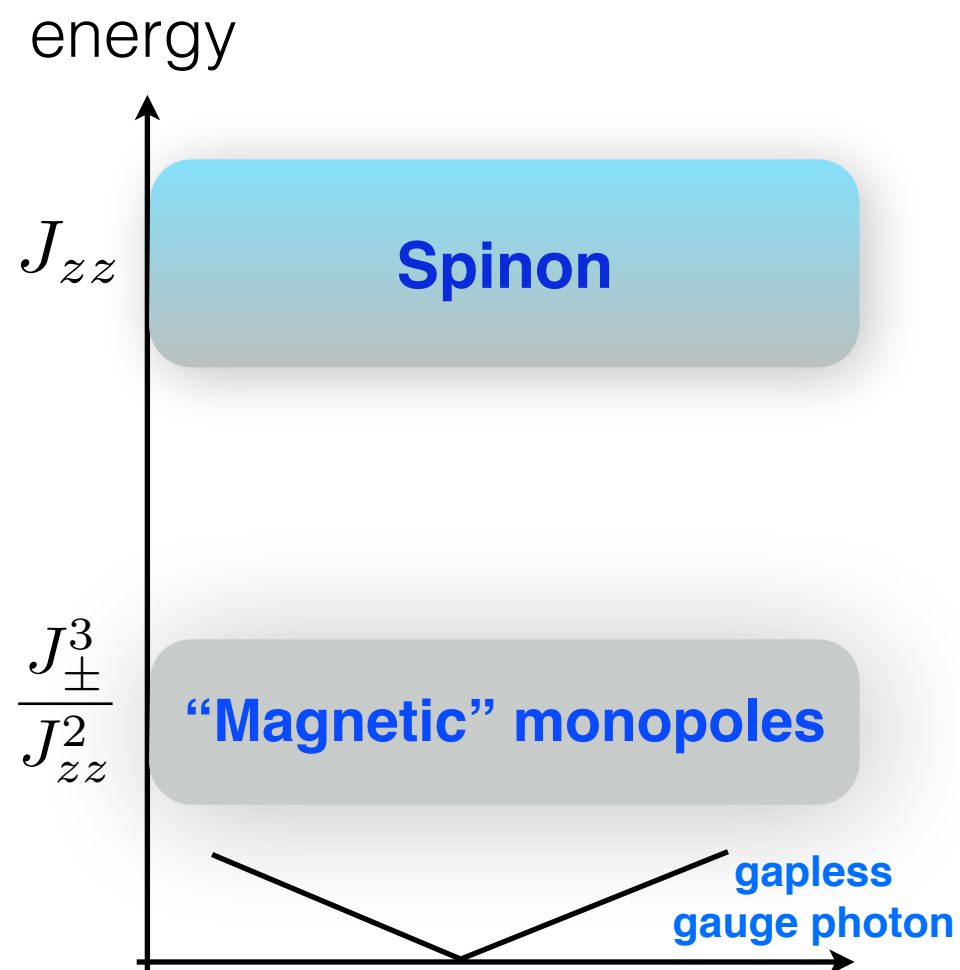
2. In contrast, quantum spin ice is a new quantum phase of matter.

3. To obtain quantum spin ice, one simply adds quantum spin flip terms to the Ising Hamiltonian.

4. The quantum term allows the system to tunnel from one spin ice state to another.

5. As long as the term is not too large, the system is disordered, and the ground state is quantum spin ice.

Emergent Quantum Electrodynamics



Figs from Mo

Spinon deconfinement

Emergent electric field

$$S^z \sim E$$

Emergent vector potential

$$S^{\pm} \sim e^{\pm iA}$$

as quantum spin ice is a disordered state, there is no long range order, it is a new phase of matter and cannot be described in the Landau's paradigm of symmetry breaking.

the right description of quantum spin ice is fractionalization and emergence of emergent gauge structure.

there are 3 elementary excitations: spinon, gauge photon, and magnetic monopole. it is not a Goldstone boson, which is associated with symmetry breaking, there is no symmetry breaking of emergent gauge structure.

there are deconfined spinons, which can create 2 spinons, you can flip the further spins, at an arbitrary distance, it is only a string of spinons. in the image there is a string connecting two spinons, the spin is strongly fluctuating, the spinons are deconfined.

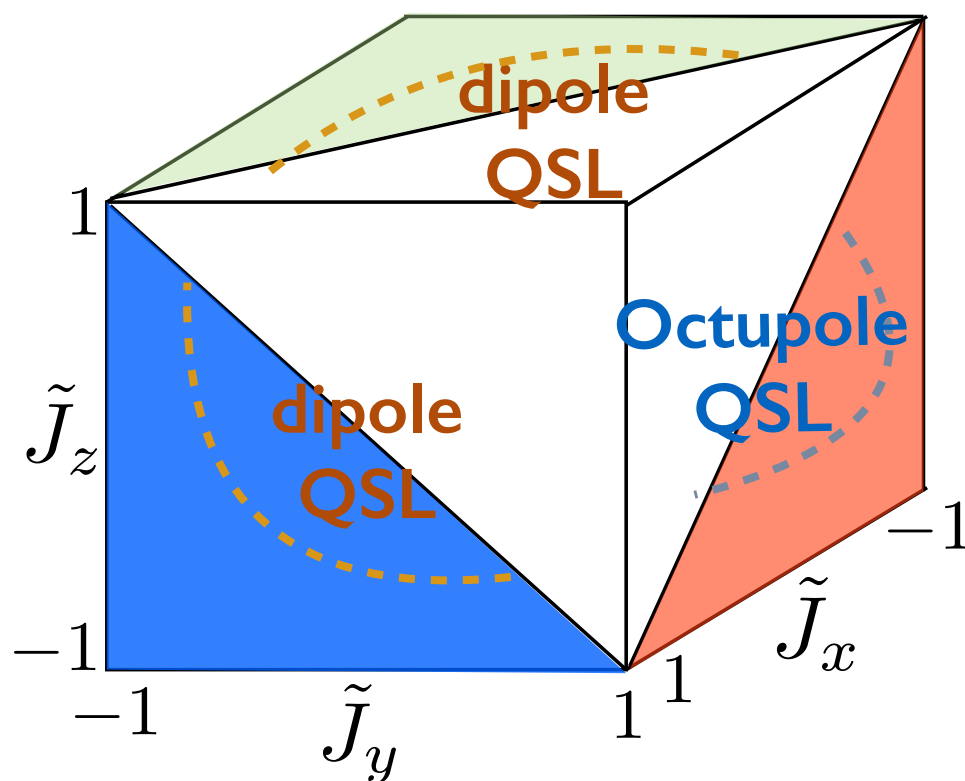
the 3rd is magnetic monopole, which is a defect of the emergent U(1) gauge field.

It is an exotic phase of matter, with properties as topological order.

XYZ model is the generic model that describes the interaction between DO doublets.

$$H_{\text{XYZ}} = \sum_{\langle ij \rangle} \mathcal{J}_x \tau_i^z \tau_j^z + \mathcal{J}_y \tau_i^y \tau_j^y + \mathcal{J}_z \tau_i^z \tau_j^z$$

unlike XXZ model, XYZ model is richer



3D phase diagram

Each component (not just S_z) can be emergent electric field, depending on the parameters !

Study phase on a cube: $-1 \leq \tilde{J}_{x,y,z} \leq 1$.

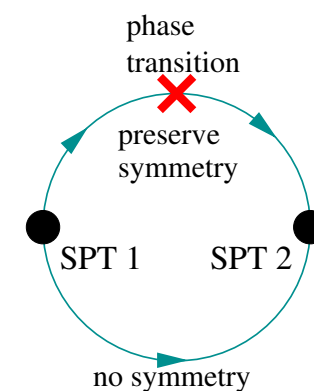
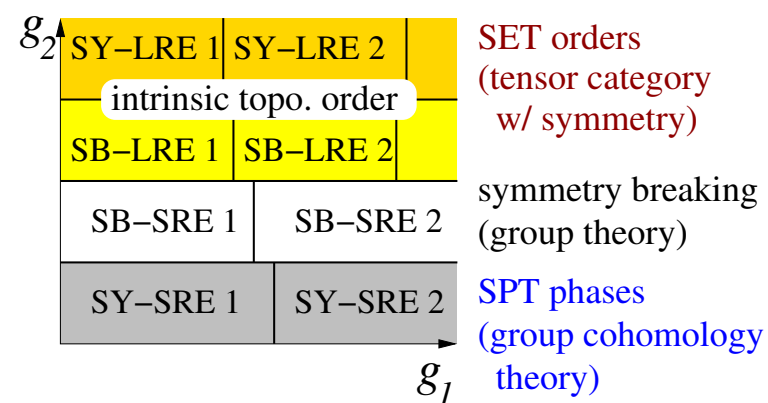
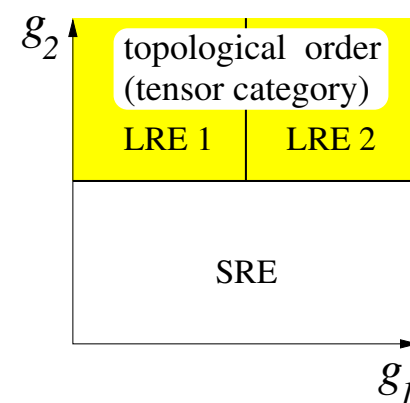


Xiaogang Wen

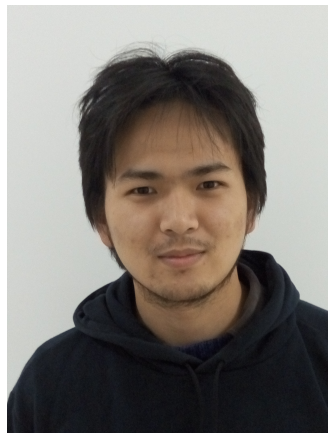
Gapped phases w/ symmetry \rightarrow SET and SPT phases

- there are **LRE symmetric states** \rightarrow **S**ymm. **E**nriched **T**opo. phases
 - **100**s symm. spin liquid through the **PSG** of topo. excit. Wen 02
 - **8** trans. symm. enriched **Z_2** topo. order in 2D, **256** in 3D Kou-Wen 09
 - **1000,000**s symm. **Z_2** spin liquid through $[\mathcal{H}^2(SG, Z_2)]^2 \times$ Hermle 12
 - Classify SET phases through $\mathcal{H}^3[SG \times GG, U(1)]$ Ran 12
- there are **SRE symmetric states** \rightarrow many different phases

We may call them **symmetry protected trivial (SPT)** phase



- Control spinons in a quantum spin ice $U(1)$ quantum spin liquid



Yao-Dong Li
(Fudan)

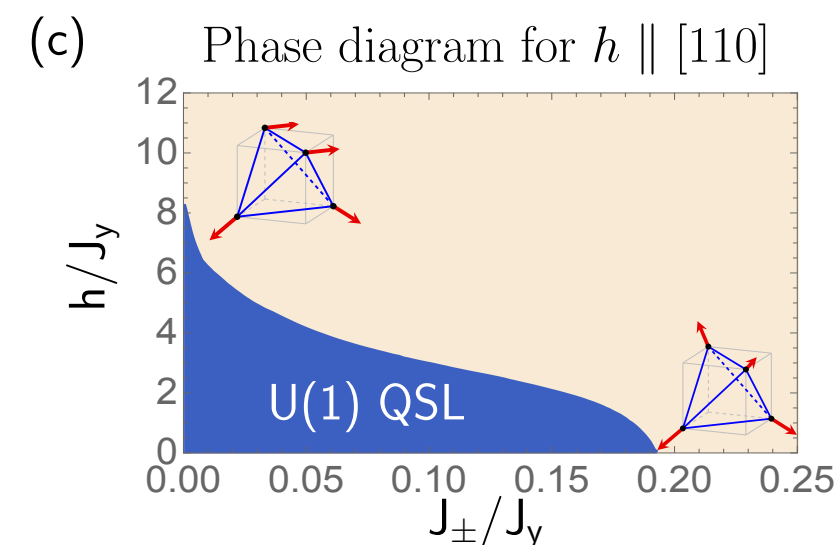
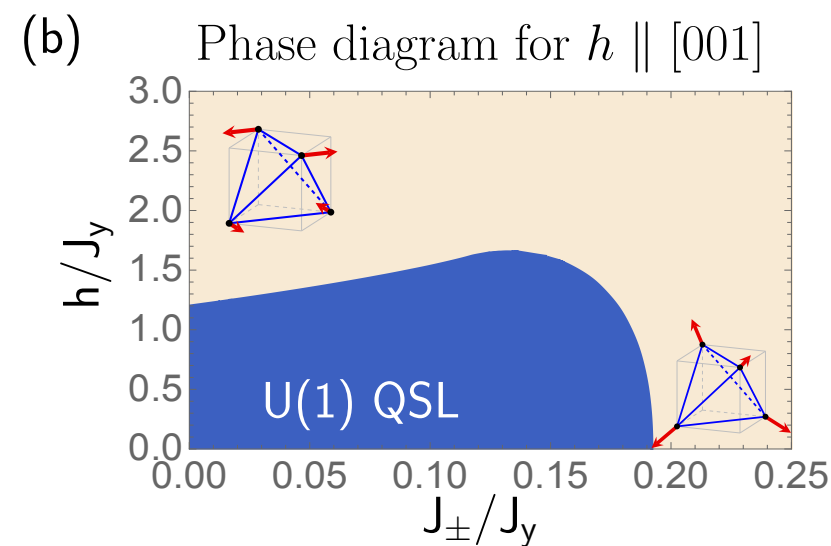
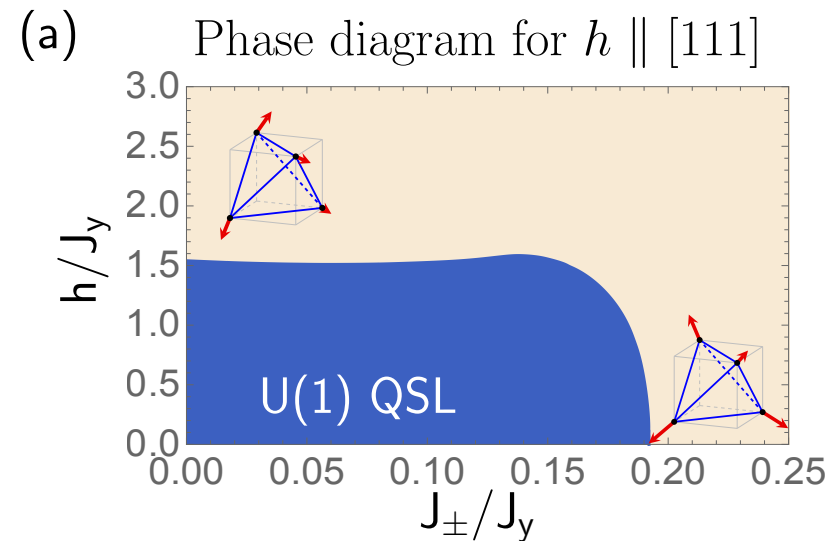
arxiv 1607

Field-driven Higgs transition for octupolar U(1) QSL

How to tell if Ce2Sn2O7 is an octupolar U(1) QSL or not ?

The idea to use a little knob that could simply lead to some clear experimental consequence, very much like the isotope effect of BCS superconductors.

Here we apply external magnetic field, and expect a field-driven Higgs transition to magnetic ordering as the **field only couples to the matter field** (spinons).

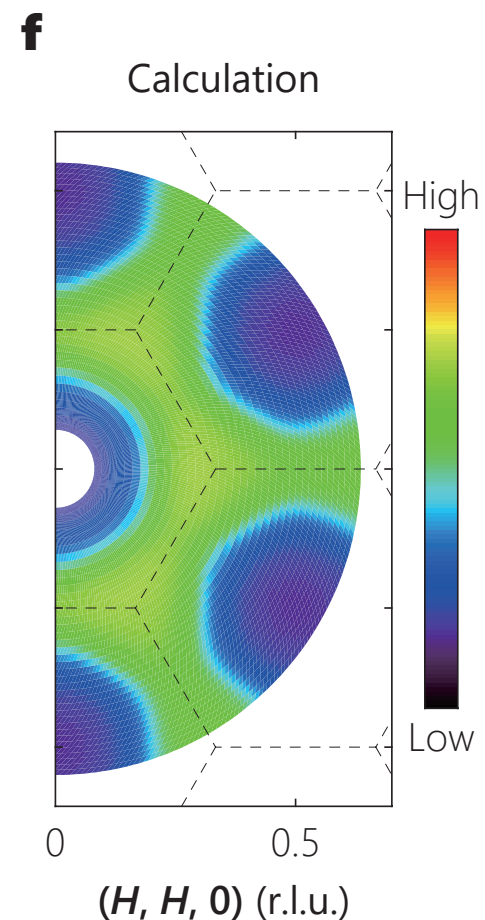
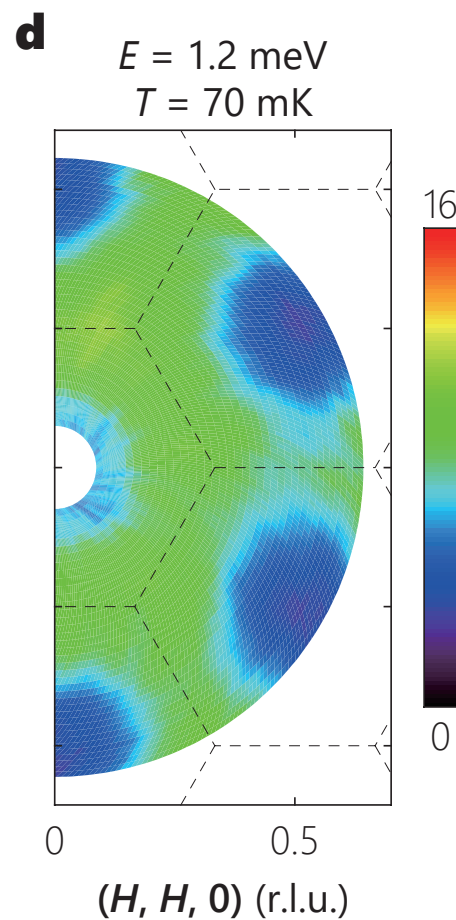
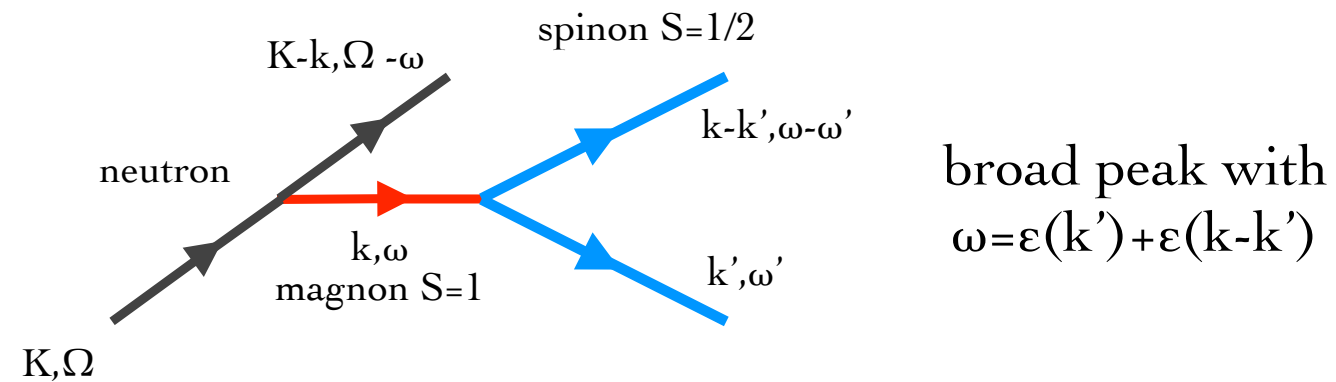


$$H_{\text{sim}} = \sum_{\langle ij \rangle} J_y \tau_i^y \tau_j^y - J_{\pm} (\tau_i^+ \tau_j^- + h.c.) - \sum_i h (\hat{n} \cdot \hat{z}_i) \tau_i^z, \\ \tau_i^{\pm} = \tau_i^z \pm i \tau_i^x$$

without losing generaliz

Higgs transition is very
minor effect in super

Inelastic neutron scattering and spinon continuum

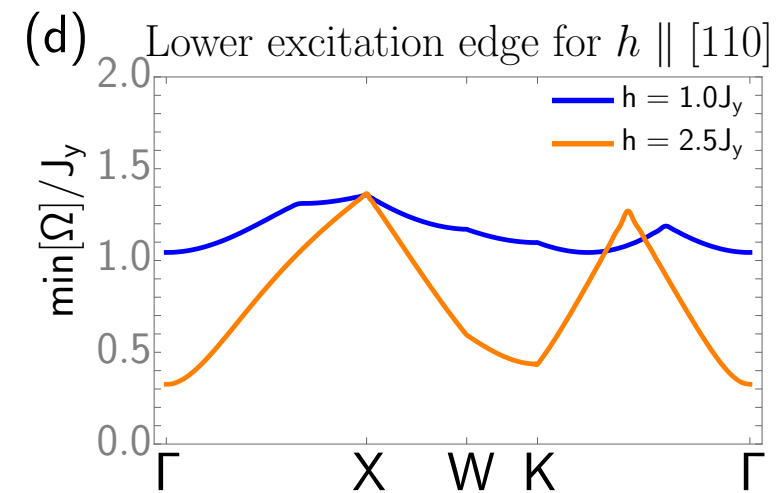
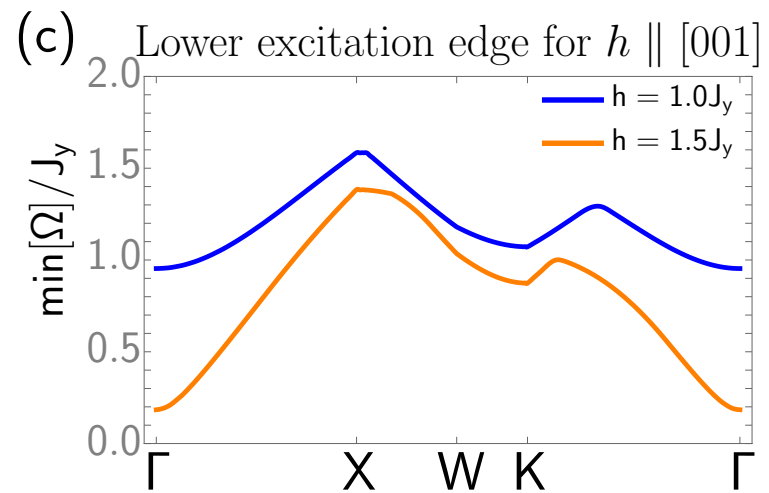
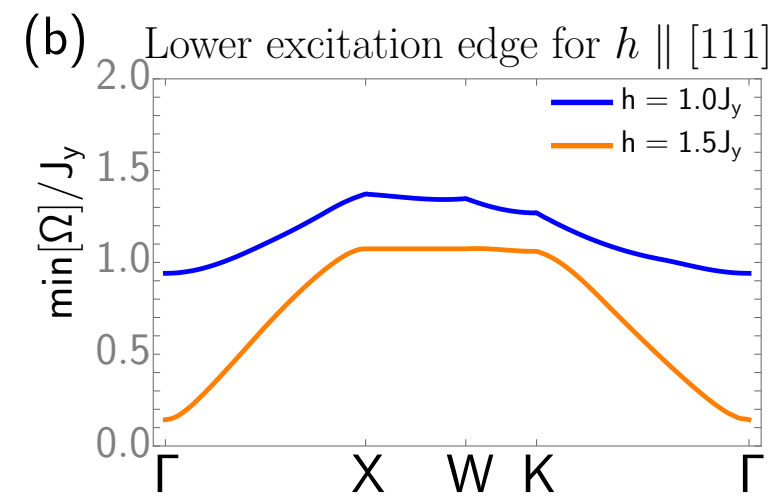
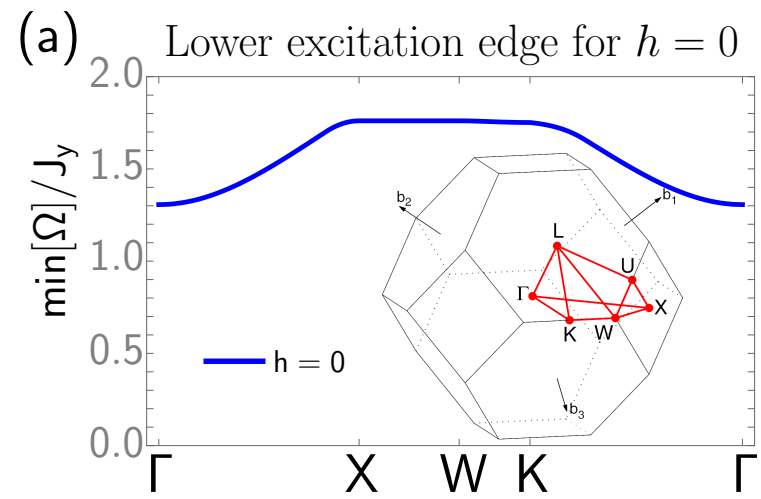


Spinon continuum
in YbMgGaO₄
(today's arXiv)

Lower excitation edge

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2,$$

$$\Omega(\mathbf{q}) = \omega_i(\mathbf{k}_1) + \omega_j(\mathbf{k}_2),$$



Neutron scattering and thermal transport

Different U(1) QSLs	Heat capacity	Inelastic neutron scattering measurement
Octupolar U(1) QSL for DO doublets	$C_v \sim T^3$	Gapped spinon continuum
Dipolar U(1) QSL for DO doublets	$C_v \sim T^3$	Both gapless gauge photon and gapped spinon continuum
Dipolar U(1) QSL for non-Kramers' doublets	$C_v \sim T^3$	Gapless gauge photon
Dipolar U(1) QSL for usual Kramers' doublets	$C_v \sim T^3$	Both gapless gauge photon and gapped spinon continuum

Thermal transport

see both contribution, but there is a big separation of energy scales in spinon and gapless photons.

Material survey: other DO doublet systems

Our doublet can potentially be realized for any Kramers spin moment with $J > 1/2$.

Two well-known systems:

- Pyrochlores $A_2B_2O_7$,

e.g. ,

$Nd_2Ir_2O_7$, $Nd_2Sn_2O_7$, $Nd_2Zr_2O_7$, etc

$Dy_2Ti_2O_7$,

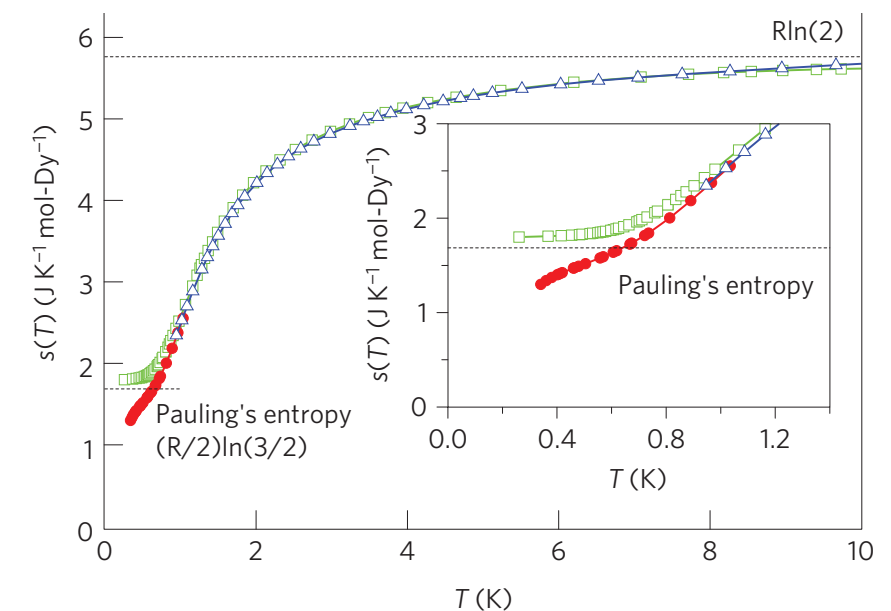
$Cd_2Os_2O_7$, etc

$Ce_2Sn_2O_7$,

- Spinel AB_2X_4 , B=lanthanide?

e.g. $CdEr_2Se_4$

$CdYb_2S_4$



Prof Gaulin's group, $Dy_2Ti_2O_7$, Nat Phys, 2013

Conclusion

- We propose a new doublet dubbed “dipole-octupole” doublet.
- We propose a generic XYZ model for our new doublet.
- This XYZ model supports both exotic (octupolar) order and symmetry enriched U(1) quantum spin liquid (quantum spin ice) ground states.
- There exist a large class of materials (not just pyrochlore, **any other lattices with the same point group**) that can support such doublets.
- The remarkable properties of the doublet allows a direct comparison between numerics and experiments. We propose a way to detect the consequence of symmetry enrichment.